ECE 488: Compound Semiconductors

M, W, F 11:00 – 11:50, 3013 ECEB
Professor John Dallesasse
2114 Micro and Nanotechnology Laboratory
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E-mail: jdallesa@illinois.edu
Office Hours: Tuesday 13:00 – 14:00
Lecture 34: November 11th, 2016
Assignments

• Reading from “Compound Semiconductors and Devices – An Introduction”
  – Mon 11/7: §’s 8.4, 8.4.1, 8.4.2, 8.4.3, 8.5
  – Fri 11/11: §’s 9.3, 9.3.1, 9.3.2

• Homework: Posted Friday 11-3, Due 11-11

• Quiz: Tentative Date is Wednesday Nov. 16th

• Next Week: Tuesday Office Hours from 3-4 Instead of 1-2
Today’s Agenda

• Superlattices and Minibands
• Franz-Keldysh Effect
• Optical Properties of Compound Semiconductors
## Tentative Schedule [2]

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<th>SEP 26: Doping and Deep Levels</th>
<th>SEP 28: The Fermi Integral, Free Carrier Concentration, Surface States</th>
<th>SEP 30: III-V Semiconductor Lattice Constant and Bandgap</th>
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<td>OCT 3: III-N and Group IV Semiconductors</td>
<td>OCT 5: Crystal Growth, Phase Diagrams</td>
<td>OCT 7: Midterm Exam (Tentative)</td>
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<td>OCT 10: Energy Band Alignment, Model-Solid Theory</td>
<td>OCT 12: Strained Layer Structures</td>
<td>OCT 14: Strain Effects on Band Edges</td>
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<td>OCT 24: Realistic Finite Quantum Wells</td>
<td>OCT 26: Superlattices and Minibands</td>
<td>OCT 28: Heterostructures in Electric Fields and the Franz-Keldysh Effect **</td>
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<td>NOV 18</td>
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<td>NOV 28</td>
<td>Heterojunction Bipolar Transistors (HBTs)</td>
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<td>NOV 30</td>
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**Guideline Only: Subject to Change**
Superlattices and Minibands

Continued
7.4.2. Resonant tunneling through double barriers:

- In a single QW, the energy eigen values are fixed and can be precisely determined. These energy states are called ‘bound states’.
- In a double-barrier system, where the two barriers confining the QW have identical thickness. If the barriers are thick, the energy eigen values of electrons within the QW are fixed. With a reduced barrier thickness, electrons in a bound state of the QW can tunnel through barriers with a finite probability. Thus, electrons are no longer ‘bound’ to the QW states. From the uncertainty principle, the associated energy level also becomes blurred within a range of \( \hbar/\tau \), where \( \tau \) is the lifetime of an electron staying in a QW.
- The electron transmission probability \( T \) of a double-barrier system is the product of two individual barriers \( T_L T_R \). If \( T_L \neq T_R \), \( T << 1 \). On the other hand, for \( T_L = T_R \), the product of \( T_L T_R \) can reach a maximum value of unity. This is called the ‘resonant tunneling’.
Analogy: Fabry-Perot Cavity

- Fabry-Perot optical cavity treatment of double barrier structures:
  The electron bouncing back and forth between two identical barriers can be seen as light trapped in a Fabry-Perot cavity.

The mirror transmission and reflection coefficients are $t_i$ and $r_i$, respectively. The amplitude of the initial wave incident from outside the cavity is one and there is a phase factor $\exp(ika)$ involved during each bounce. The total transmission of the wave through the right mirror is

\[
t = t_L \exp(ika)t_R + t_L \exp(ika)r_R \exp(ika)r_L \exp(ika)t_R \\
+ t_L \exp(ika)r_R \exp(ika)r_L \exp(ika)r_R \exp(ika)r_L \exp(ika)t_R + \ldots \\
= t_L \exp(ika)t_R \left[1 + r_Lr_R \exp(i2ka) + r_L^2r_R^2 \exp(i4ka) + \ldots \right]
\]
Resonant Tunneling (2)

This is a geometric series of \( x = r_L r_R \exp(i 2ka) \), and \(|x| < 1\).

\[
t = t_L \exp(ika) t_R \left[ 1 + x + x^2 + x^3 + \ldots \right] = \frac{t_L t_R}{1 - r_L \exp(i 2ka) r_R} \exp(ika)
\]

The corresponding flux transmission coefficient \( T = |tt^*| \) is

\[
T = \frac{T_L T_R}{\left(1 - \sqrt{R_L R_R} \right)^2 + 4 \sqrt{R_L R_R} \sin^2 \left(\frac{\phi}{2}\right)}
\]

Here we replace the complex reflection amplitude with the polar form such as \( r_L = |r_L| \exp(i \theta_L) \). The phase angle \( \phi = (\theta_L + \theta_R + 2ka) \), and \( R_i = 1 - T_i \). Since \( R_i \) and \( T_i \) are slow varying functions, \( T \) reaches a maximum when \( \sin \phi = 0 \). This requires \( \phi = 2n \pi \), which are the resonant states. Since both \( T_L \) and \( T_R \) are small, by expanding \( \sqrt{R_L} \) and \( \sqrt{R_R} \) with binomial series, we have

\[
T = T_{pk} = \frac{T_R T_L}{\left(1 - \sqrt{R_L R_R} \right)^2} \approx \frac{4T_R T_L}{(T_R + T_L)^2} \approx 1
\]

\[
\left(\sqrt{R} = \sqrt{1 - T} = 1 - T/2 - T^2/8 - \ldots \right) \approx 1 - T/2 \text{ for } T \ll 1
\]

\[
\left[1 - \left(1 - \frac{T_R}{2}\right) \left(1 - \frac{T_L}{2}\right)\right]^2 = \left[1 - \left(1 - \frac{T_R}{2} - \frac{T_L}{2} + \frac{T_R T_L}{4}\right)\right]^2 \approx \frac{1}{4} (T_R + T_L)^2
\]

Therefore, in a double-barrier QW system, electrons can easily tunnel through barriers without loss.
Resonant Tunneling: Transmission

The flux transmission coefficient of a double-barrier system with barrier height of $0.3eV$ and thickness of $50\text{Å}$ is shown below. The dashed line is the single barrier value of $T$. At each resonant energy, the transmission coefficient reaches unity in a finite energy range and electrons can move freely into and out of the QW.
Minibands in a Superlattice

- When stacking alternating thin layers of two different semiconductors, a series of identical QW/barrier structure is formed. The periodic band edge energy variation of the structure creates an artificial 1D lattice constant in the epilayer growth direction and is called a ‘superlattice’.
- Similar to the double-barrier QW structure, an electron can tunnel from one well to its neighbors in a SL structure. The resonant states with a finite energy distribution are replacing the sharp bound states of a QW. These allowed energy bands are called ‘minibands’.
- Due to the 1D nature of SL, the Kronig-Penney model is suitable for the understanding of the SL with minor modifications.
  - $V_o \rightarrow \Delta E_c$ or $\Delta E_v$
  - $m_o \rightarrow m_w^*$ and $m_b^*$
  - $(d\psi_w/dz)/m_w^* = (d\psi_b/dz)/m_b^*$ at the boundary
Energies of Band Edges

The solution for $E < V_o$ is obtained as

$$\frac{1}{2} \left( \eta - \frac{1}{\eta} \right) \sin(\alpha a) \sinh(\gamma b) + \cos(\alpha a) \cosh(\gamma b) = \cos q(a + b)$$

where

$$\eta = \frac{\gamma m_w^*}{\alpha m_b^*}; \quad \alpha = \sqrt{\frac{2m_w^* E}{\hbar^2}}; \quad \gamma = \sqrt{\frac{2m_b^*(\Delta E_C - E)}{\hbar^2}}$$

and $q$ is the wave vector of the SL $[-\pi/(a+b) < q < \pi/(a+b)]$.

The allowed GaAs SL bands as a function of barrier thickness with a well depth of 0.3$eV$ and width of 50Å are shown below.
Density of States in a Superlattice

A SL structure contains individual but identical QWs with a fixed linear periodicity. The SL DOS can be seen as a linear combination of DOS from QWs ($D_{2D}$) and a linear periodic lattice ($D_{1D}$).

\[
D_{SL} = \frac{1}{2} \int_{-\infty}^{\infty} D_{1D}(E)D_{2D}(E)dE = \frac{m^*}{2\pi\hbar^2} \int_{-\infty}^{\infty} D_{1D}(E)dE
\]

The factor of (1/2) avoids double counting the spin. Now, we need to evaluate $D_{1D}$. Assume a linear 1D lattice of a total length $L$.

\[
k = \frac{n\pi}{L} = \sqrt{\frac{2mE}{\hbar^2}}; \quad n = \frac{L}{\pi} \sqrt{\frac{2mE}{\hbar^2}}
\]

\[
\Rightarrow D_{1D} = \frac{1}{L} \frac{dn}{dE} = \frac{1}{\pi\hbar} \sqrt{\frac{2m}{E}}
\]

Since $E = \frac{mv^2}{2}$,

\[
\sqrt{\frac{2m}{E}} = \frac{2}{v(E)}
\]

\[
\therefore D_{1D} = \frac{2}{\pi\hbar v(E)} \quad \text{and} \quad D_{SL} = \frac{m^*}{2\pi\hbar^2} \int \frac{2}{\pi\hbar v(E)} dE
\]

\[
E = \frac{\hbar^2 k^2}{2m^*} = \frac{p^2}{2m^*} = \left(\frac{m^* v}{2m^*}\right)^2 = \frac{m^* v^2}{2}
\]
Density of States in a Superlattice (2)

Using the band edge of a tight-binding model and set \( E = (\Delta/2)(1 - \cos kc) \), where \( c = a + b \), we can solve \( v(E) \).

\[
v(E) = \frac{1}{\hbar} \frac{dE}{dk} = \frac{c\Delta}{2\hbar} \sin kc
\]

We can deduce the sine function from the cosine function of the tight-binding model.

\[
\cos kc = 1 - \frac{E}{(\Delta/2)} = \frac{(\Delta/2) - E}{(\Delta/2)} \quad \text{and} \quad \sin kc = \frac{\sqrt{(\Delta/2)^2 - [(\Delta/2) - E]^2}}{(\Delta/2)}
\]

Therefore, \( v(E) = \frac{c}{\hbar} \sqrt{\left(\frac{\Delta}{2}\right)^2 - \left(\frac{\Delta}{2} - E\right)^2} \)

\[
D_{AD}(E) = \frac{2}{\pi\hbar v(E)} = \frac{2}{c\pi} \left[\left(\frac{\Delta}{2}\right)^2 - \left(\frac{\Delta}{2} - E\right)^2\right]^{-1/2}
\]

\[
\Rightarrow \quad D_{SL} = \frac{m^*}{2\pi\hbar^2} \int \frac{2}{c\pi} \frac{dE}{\sqrt{(\Delta/2)^2 - [(\Delta/2) - E]^2}} = \frac{m^*}{c\pi^2\hbar^2} \sin^{-1}\left[\frac{E - (\Delta/2)}{(\Delta/2)}\right]_{-\infty}^{E}
\]

\[
= \frac{m^*}{c\pi^2\hbar^2} \left\{ \sin^{-1}\left[\frac{E - (\Delta/2)}{(\Delta/2)}\right] - \sin^{-1}(-\infty) \right\}_{-\pi/2}^{\pi/2}
\]
Superlattice Density of States (2)

\[ D_{SL} = \frac{m^*}{c\pi\hbar^2} \left\{ \frac{1}{2} + \frac{1}{\pi} \sin^{-1} \left[ \frac{E - (\Delta/2)}{(\Delta/2)} \right] \right\} \quad 0 < E < \Delta \]

- Thick barriers: The 1D-SL contribution is weak. It maintains a step-like DOS as in QWs.
- Thin barriers: Communication of electrons between wells increases and broadens the steps in DOS into arcsine of width \( \Delta_i \).

![Graph showing the density of states for QW and 1D-SL](image)
Heterostructures in Electric Fields
The Franz-Keldysh Effect: Bulk Semiconductors

The absorption coefficient of a semiconductor in an electric field changes with the applied electric field. This effect was observed independently by Franz and Keldysh. The physical origin of this phenomenon can be understood by solving the 1D Schrödinger equation.

When an electric field intensity $F$ is applied to a semiconductor along $z$-direction, the wave function has the form of

$$\left(-\frac{\hbar^2}{2m} \frac{d^2}{dz^2} + eFz\right)\psi(z) = E\psi(z)$$

This is similar to the triangular potential well case and has the solutions of

$$\psi(z) = A_i \left(\frac{eFz - E}{E_o}\right) \quad \text{and} \quad E_o = \left[\frac{(eF\hbar)^2}{2m}\right]^{1/3} = eFL$$

The major differences between these two cases are:

- In triangular QWs, the energy eigen values are quantized and have discrete values;
Franz-Keldysh Effect (Bulk)

- In bulk materials, there is no potential barrier on the low potential side. Therefore, there is no restriction of selecting $E$, and the solutions of $E$ are numerous and continuous.
- In the bulk, the interaction of the incident wave and the reflection wave generated by the potential barrier form a standing wave rather than propagating. The undulation becomes more rapidly with increasing distance from the potential barrier surface.
Transitions Under High Fields

- Band-to-band transitions:
  - Since the decaying wave extends into the forbidden gap, the conduction band decaying wave overlaps with that from the valence band under high fields. The optical absorption corresponds to an energy smaller than the band gap ($\Delta E < E_g$).

- Under an even stronger electric field intensity, one of the oscillating part of the wave function overlap with the the other oscillation function or the decaying wave function. The transition energy is larger than the band gap energy. In addition, the absorption will show an oscillating nature due to the field-dependent phase change in wave functions.
Example: GaAs Under High Field

Effect of an electric field on optical absorption in GaAs

Quantum-Confined Stark Effect

- Stark effect:
  An electric field tends to orient the elliptical orbits of electrons so that the center of gravity of the ellipse and the focus of the ellipse (the nucleus) are aligned with the electric field $F$. The circular orbits like the $s$-orbitals are not affected by the electric field. But the excited states, like $p$-states, have elliptical orbits and will be affected.

  ![Diagram of elliptical orbit and electric field](image)

  The energy shift of the electron state due to the formation of the dipole moment is $\Delta E \sim \pm qdF$. This is the first-order (linear) Stark effect.

- QCSE:
  Consider the ground state in a QW under zero field, the electron wave peak and the hole wave peak are aligned in the middle of the QW. Under an applied electric field, the wave function peaks of electrons and holes move to opposite direction.
Quantum Confined Stark Effect

(a) constant potential

(b) in electric field

- Under an electric field $F$, the relative displacement of wave function peaks of electrons and holes in a QW is $\langle x \rangle$, which produces an electric dipole moment $p = -q \langle x \rangle$. This induced dipole moment reduces the QW transition energy to $E_{\text{QCSE}} < E_{\text{QW}}$. This is called the quantum confined Stark effect (QCSE).

- However, due to the misaligned electron and hole wave function peaks, the strength of optical absorption and emission is reduced.

- The tunneling probability for low barrier holes increases rapidly with increasing field intensity $F$. 
Optical Absorption

- Sample structure: 93Å GaAs QW sandwiched by AlGaAs superlattice barriers.
- For incident optical field parallel to the sample surface ($\parallel QW$), both HH and LH absorption peaks are observed. (TE mode)
- For incident field perpendicular to the sample surface ($\perp QW$), only LH absorption peak is observed. (TM mode)
- The absorption peak makes a red-shift with increasing the applied electric field intensity and shows reduced absorption strength.

Self Electro-Optic Effect Devices

Application – Self-Electro-optic Effect Device (SEED)
Agenda for Next Class

• Optical Properties of Heterstructures
Thank You!
## Final Exam

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<td>66375</td>
<td>12/12/2016</td>
<td>M</td>
<td>7:00 PM</td>
<td>10:00 PM</td>
<td>3017 Electrical &amp; Computer Eng Bldg</td>
<td>Extra Space</td>
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For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.
Common Semiconductors

Fig. 21.4. Room-temperature bandgap energy versus lattice constant of common elemental and binary compound semiconductors.

“Italic” = indirect gap
“Roman” = direct gap
○ hexagonal structure
□ cubic structure

E. F. Schubert
Light-Emitting Diodes (Cambridge Univ. Press)
www.LightEmittingDiodes.org
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Website:
https://courses.engr.illinois.edu/ece488/
Course Objectives
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• Develop a working knowledge of compound semiconductor materials and devices
• Provide a foundation for future advanced physical electronics courses
• Provide basic device knowledge to support a career in wireless communications or photonics
• Provide sufficient background such that you can begin to read and understand the literature on compound semiconductor materials and devices
Course Outline
Course Outline

• Review of semiconductor fundamentals
  – Elementary quantum mechanics
  – Atomic bonding and crystal structures
  – Electronic band structures of solids
• Compound semiconductor materials
  – Compound semiconductor crystals
  – Material technologies
• Properties of heterostructures
  – Basic heterostructure properties
  – Electrical properties of heterostructures
  – Optical properties of heterostructures
• Heterostructure devices
  – High-speed electronic devices
  – Semiconductor lasers
  – New device development
• Review of quantum, mechanical basics including wave-particle duality, Schroedinger wave equation, one-dimensional free and bounded particles in quantum wells
• Introduction to compound semiconductor crystals, structural and electrical properties, free carrier concentration and Fermi-Dirac integral, III-V alloys
• Phase equilibrium, growth of bulk crystals and phase equilibrium, liquid phase epitaxy, vapor phase epitaxy, metalorganic chemical vapor deposition, molecular beam epitaxy
• Basic heterostructure properties, energy band alignment models, strain effect on the bandgap energies, abrupt p-n heterojunction in equilibrium, heterojunction under bias
• Electronic properties of real quantum wells, potential barrier and tunneling, superlattices and miniband, quantum wells in electric fields, modulation doping and two-dimensional electron gas
• Optical properties of dielectrics, absorption, radiative transitions - Einstein relations, stimulated emission, absorption and emission rates in semiconductors, transitions in degenerated semiconductors, nonradiative recombination processes
• Metal-semiconductor field-effect transistors, pseudomorphic high-electron mobility transistors, heterojunction bipolar transistors, transfer electron devices, resonant tunneling devices
• Photodetectors, solar cells, light-emitting diodes (LEDs), dielectric waveguide and heterostructure laser theories, quantum well lasers, distributed feedback lasers, vertical cavity surface emitting lasers
Prerequisites

• ECE340 or equivalent basic semiconductor course
• Physics background – Basic modern physics
• Math background – differential equations
## Tentative Schedule [1]

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<td>AUG 29: Potential Steps, Coulomb Well (Hydrogen Atom), Atomic Bonding</td>
<td>AUG 31: Crystal Structures, Diffraction</td>
<td>SEP 2: Reciprocal Space, Diffraction Condition</td>
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<tr>
<td>SEP 5: LABOR DAY NO CLASS</td>
<td>SEP 7: The Brillouin Zone, Band Structures, Density of States</td>
<td>SEP 9: Bloch Theorem, Empty Lattice Model</td>
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<td>SEP 12: Band Gaps</td>
<td>SEP 14: Kronig-Penny Model</td>
<td>SEP 16: Effective Mass, Bloch Oscillations, Band Structure</td>
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Grading and Policies
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<td>Final Exam</td>
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Homework:  
• Due 1 week after assigned, due in class, no late homework accepted  
Quizzes:  
• 2 quizzes, dates will be announced ahead of time, 20 minutes  
Exam(s):  
• Calculator allowed  
• 8.5 X 11, hand-written, double-sided formula sheet  
Key Points:  
• Come to class  
• Do your homework  
• If you’re having problems attend office hours
Other Comments

• Ask questions if you have them
• Don’t miss quizzes, exams, or homework
• Turn off your cell phones
• No video recording or photography in class
• Include name and NetID on all documents turned in for credit
Class notes (required) can be purchased from the ECE Supply Center.
Additional reading materials will be distributed in class or through the course website.
Reference for further reading (NOT required):

- **Solid state physics:**
  - C. Kittel, *Introduction to solid state physics* (any edition), John Wiley

- **Semiconductor physics and devices:**
  - S.L. Chuang, *Physics of Semiconductor Devices*

- **Quantum wells and heterostructures:**

- **Compound semiconductor materials:**