ECE 488: Compound Semiconductors

M,W,F 11:00 – 11:50, 3013 ECEB
Professor John Dallesasse
2114 Micro and Nanotechnology Laboratory
Tel: (217) 333-8416
E-mail: jdallesa@illinois.edu
Office Hours: Tuesday 13:00 – 14:00
Lecture 33: November 9th, 2016
Assignments

• Reading from “Compound Semiconductors and Devices – An Introduction”
  – Mon 11/7: §’s 8.4, 8.4.1, 8.4.2, 8.4.3, 8.5
  – Fri 11/11: §’s 9.3, 9.3.1, 9.3.2
• Homework: Posted Friday 11-3, Due 11-11
• Quiz: Tentative Date is Wednesday Nov. 16th
Today’s Agenda

• Hole States in Quantum Wells
• Strained Wells
• Superlattices and Minibands
<table>
<thead>
<tr>
<th>SEP 26: Doping and Deep Levels</th>
<th>SEP 28: The Fermi Integral, Free Carrier Concentration, Surface States</th>
<th>SEP 30: III-V Semiconductor Lattice Constant and Bandgap</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCT 3: III-N and Group IV Semiconductors</td>
<td>OCT 5: Crystal Growth, Phase Diagrams</td>
<td>OCT 7: Midterm Exam (Tentative)</td>
</tr>
<tr>
<td>OCT 10: Energy Band Alignment, Model-Solid Theory</td>
<td>OCT 12: Strained Layer Structures</td>
<td>OCT 14: Strain Effects on Band Edges</td>
</tr>
<tr>
<td>OCT 24: Realistic Finite Quantum Wells</td>
<td>OCT 26: Superlattices and Minibands</td>
<td>OCT 28: Heterostructures in Electric Fields and the Franz-Keldysh Effect</td>
</tr>
</tbody>
</table>

**Guideline Only: Subject to Change**
<table>
<thead>
<tr>
<th>Date</th>
<th>Topic</th>
<th>Date</th>
<th>Topic</th>
<th>Date</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCT 31:</td>
<td>Optical Properties of Dielectric Media</td>
<td>NOV 2:</td>
<td>Absorption in Semiconductors</td>
<td>NOV 4:</td>
<td>Transitions Between Discrete States</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NOV 7:</td>
<td>Radiative and Non-Radiative Transitions Between Bands</td>
<td>NOV 9:</td>
<td>Introduction to Heterojunction Devices, MESFETs</td>
<td>NOV 11:</td>
<td>Modulation Doping</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NOV 14:</td>
<td>High Electron Mobility Transistors (HEMTs)</td>
<td>NOV 16:</td>
<td>High Electron Mobility Transistors (HEMTs)</td>
<td>NOV 18:</td>
<td>GaN High Electron Mobility Transistors; NOV 21-25: Thanksgiving</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NOV 28:</td>
<td>Heterojunction Bipolar Transistors (HBTs)</td>
<td>NOV 30:</td>
<td>Heterojunction Bipolar Transistors</td>
<td>DEC 2:</td>
<td>Heterostructure Lasers</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DEC 5:</td>
<td>Heterostructure Lasers</td>
<td>DEC 7:</td>
<td>Photodiodes and Solar Cells; Last Lecture</td>
<td>FINAL EXAM:</td>
<td>Per Registrar’s Office</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Guideline Only: Subject to Change**
Hole Energy Levels in Quantum Wells
Hole Energy Levels in Quantum Wells

Due to degeneracy in the valence band, the energy functions are much more complex in QWs. In bulk semiconductors, the hole energy values near the zone center can be described as

$$E(k) = E_v - \frac{\hbar^2 k^2}{2m_h^*}; \quad m_h^* = m_{lh}^* \text{ or } m_{hh}^*$$

In QWs, the 1D confinement of wave functions destroys the symmetry and leads to the followings.

- LH and HH bands are decoupled:
  The origin of the valence bands is the $p$-like orbital electrons with a strong directionality. The barrier in $z$-direction further restrict the flow of carriers. Thus the bound HH state, which has $p$-orbitals oriented in $xy$-plane, sits deeper in the QW with a lower energy than the LH state.
Hole Energy Levels (2)

- In a QW, for HH band, the mass looks even heavier due to the restriction along z-axis, which means the HH state sits deep in QW.
- HH and LH band splits.

The LH has a heavier \(xy\)-plane mass and the kinetic energy rises slowly with \(k\).

The HH has a lighter mass in \(xy\)-plane and the kinetic energy rises rapidly with \(k\).
LH and HH bands are strongly mixed: The orientation of $p$-orbital electrons for HH and LH is different and perpendicular to each other inside the QW. The LH has the $p$-orbitals oriented in $z$-direction and the HH state has $p$-orbitals oriented in $xy$-plane and sits deeper in the QW. Therefore, the HH and LH bands are decoupled.

In the $xy$-plane, within the QW, the HH has a lighter in-plane mass and the kinetic energy, $\hbar k$, rises rapidly with $k$. On the other hand, the LH has a heavier in-plane mass and the kinetic energy rises slowly with $k$. (See figure below).

The HH and LH bands are strongly mixed and cross over at some value of $k$.

In the direction of quantization, the HH is heavier and the LH is lighter.
Hole Energy Levels (4)

- Modified effective masses along [001] (z-direction, perpendicular to QW) can be expressed in term of Lüttinger parameters.

\[ m_{lh}^z = \frac{m_o}{\gamma_1 + 2\gamma_2}; \quad m_{hh}^z = \frac{m_o}{\gamma_1 - 2\gamma_2} \]

- Modified effective masses along [110] (x- and y-direction) can be expressed in term of Lüttinger parameters.

\[ m_{lh}^{xy} = \frac{m_o}{\gamma_1 - \gamma_2}; \quad m_{hh}^{xy} = \frac{m_o}{\gamma_1 + \gamma_2} \]

- Example of the valence band diagram of a 100Å GaAs-Al_{0.3}Ga_{0.7}As QW. The subbands are highly deviated from the idea parabolic shape. Strong band mixing is clearly displayed.

The Luttinger parameters are related to the p-like orbitals that form the valence band.
Strained Quantum Wells

In a strained QW structure, the bandedge of barriers is unchanged. Only the bandedge of the strained QW is modified to include the strain effect.

\[ E_c = E_v^o + H' \]

\[ E_v = E_v^o + H \pm S \quad \text{for} \quad \begin{cases} HH \\ LH \end{cases} \]

The QW bandedge positions are determined from the same equations developed earlier and shown below.

\[ H = 2a_v \varepsilon_1 \left( \frac{C_{11} - C_{12}}{C_{11}} \right) \]

\[ S = b \varepsilon_1 \left( \frac{C_{11} + 2C_{12}}{C_{11}} \right) \]

\[ H' = 2a_c \varepsilon_1 \left( \frac{C_{11} - C_{12}}{C_{11}} \right) \]
Superlattices and Minibands
7.4.1. Square barrier:

A wave with energy $E$ is incident from the left and tunnel through a barrier of thickness $a$. The transmission amplitude derived in Chapter 1 is

$$t = \frac{2k_1k_2 \exp(-ik_1a)}{2k_1k_2 \cos k_2a - i(k_1^2 + k_2^2) \sin k_2a}$$

where $k_1 = \sqrt{\frac{2mE}{\hbar^2}}$ and $k_2 = \sqrt{\frac{2m(E-V_o)}{\hbar^2}}$

For $E < V_o$, the transmitted flux is

$$T = |tt^*| = \frac{4k_1^2k_2^2}{4k_1^2k_2^2 + (k_1^2 + k_2^2)^2 \sinh^2(k_2a)} = \left[1 + \frac{V_o^2}{4E(V_o - E) \sinh^2(k_2a)}\right]^{-1}$$

where $k_2 = i\kappa_2$. Since $\sinh(\kappa_2a)$ increases rapidly with $\kappa_2a$, the transmission flux remains small even for a thin barrier.
Square Barrier (2)

For $E > V_o$, the transmitted flux has the following form.

$$T = \frac{4k_1^2 k_2^2}{4k_1^2 k_2^2 + \left(k_1^2 - k_2^2\right)^2 \sin^2(k_2 a)} = \left[1 + \frac{V_o^2}{4E(E - V_o)} \sin^2(k_2 a)\right]^{-1}$$

At $E \approx V_o$, $k_2 \approx 0$, and $\sin^2(k_2 a) \approx (k_2 a)^2 = 2ma^2(E - V_o)/h^2$. Thus, the transmission is

$$T = \left[1 + \frac{ma^2 V_o}{2h^2}\right]^{-1} \ll 1$$

The transmission through a GaAs square barrier of height $V_o = 0.3eV$ and thickness $a = 10nm$ is shown below. Even for $E > V_o$, $T$ is unity only when $\sin(k_2 a) = 0$. 

![Graph showing transmission through a square barrier](image)
7.4.2. Resonant tunneling through double barriers:

- In a single QW, the energy eigen values are fixed and can be precisely determined. These energy states are called ‘bound states’.
- In a double-barrier system, where the two barriers confining the QW have identical thickness. If the barriers are thick, the energy eigen values of electrons within the QW are fixed.
  
  With a reduced barrier thickness, electrons in a bound state of the QW can tunnel through barriers with a finite probability. Thus, electrons are no longer ‘bound’ to the QW states. From the uncertainty principle, the associated energy level also becomes blurred within a range of \( \frac{\hbar}{\tau} \), where \( \tau \) is the lifetime of an electron staying in a QW.

- The electron transmission probability \( T \) of a double-barrier system is the product of two individual barriers \( T_L T_R \). If \( T_L \neq T_R \), \( T << 1 \). On the other hand, for \( T_L = T_R \), the product of \( T_L T_R \) can reach a maximum value of unity. This is called the ‘resonant tunneling’.
Analogy: Fabry-Perot Cavity

- Fabry-Perot optical cavity treatment of double barrier structures: The electron bouncing back and forth between two identical barriers can be seen as light trapped in a Fabry-Perot cavity.

The mirror transmission and reflection coefficients are $t_i$ and $r_i$, respectively. The amplitude of the initial wave incident from outside the cavity is one and there is a phase factor $\exp(ika)$ involved during each bounce. The total transmission of the wave through the right mirror is

$$t = t_L \exp(ika)t_R + t_L \exp(ika)r_R \exp(ika)r_L \exp(ika)t_R$$

$$+ t_L \exp(ika)r_R \exp(ika)r_L \exp(ika)r_R \exp(ika)r_L \exp(ika)t_R + \ldots$$

$$= t_L \exp(ika)t_R \left[1 + r_L r_R \exp(i2ka) + r_L^2 r_R^2 \exp(i4ka) + \ldots\right]$$
Resonant Tunneling (2)

This is a geometric series of $x = r_L r_R \exp(i2ka)$, and $|x| < 1$.

$$t = t_L \exp(ika) r_R \left[1 + x + x^2 + x^3 + \ldots\right] = \frac{t_L t_R}{1 - r_L \exp(i2ka) r_R} \exp(ika)$$

The corresponding flux transmission coefficient $T = |t t^*|$ is

$$T = \frac{T_L T_R}{\left(1 - \sqrt{R_L R_R}\right)^2 + 4 \sqrt{R_L R_R} \sin^2 (\phi/2)}$$

Here we replace the complex reflection amplitude with the polar form such as $r_L = |r_L| \exp(i \theta_L)$. The phase angle $\phi = (\theta_L + \theta_R + 2ka)$, and $R_i = 1 - T_i$.

Since $R_i$ and $T_i$ are slow varying functions, $T$ reaches a maximum when $\sin \phi = 0$. This requires $\phi = 2n\pi$, which are the resonant states. Since both $T_L$ and $T_R$ are small, by expanding $\sqrt{R_L}$ and $\sqrt{R_R}$ with binomial series, we have

$$T = T_{pk} = \frac{T_R T_L}{\left(1 - \sqrt{R_L R_R}\right)^2} \approx \frac{4T_R T_L}{(T_R + T_L)^2} \approx 1$$

$$\left(\sqrt{R} = \sqrt{1-T} = 1 - T/2 - T^2/8 - \ldots \approx 1 - T/2 \quad \text{for} \quad T << 1\right)$$

Therefore, in a double-barrier QW system, electrons can easily tunnel through barriers without loss.
Resonant Tunneling: Transmission

The flux transmission coefficient of a double-barrier system with barrier height of 0.3\,eV and thickness of 50\,\AA\ is shown below. The dashed line is the single barrier value of $T$. At each resonant energy, the transmission coefficient reaches unity in a finite energy range and electrons can move freely into and out of the QW.
Agenda for Next Class

• Superlattices and Minibands
• Heterostructures in Electric Fields
• The Franz-Keldysh Effect
Thank You!
# Final Exam

<table>
<thead>
<tr>
<th>Course</th>
<th>Section</th>
<th>CRN</th>
<th>Date</th>
<th>Day</th>
<th>Start Time</th>
<th>End Time</th>
<th>Room</th>
<th>Exam Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECE 488</td>
<td>C</td>
<td>66375</td>
<td>12/12/2016</td>
<td>M</td>
<td>7:00 PM</td>
<td>10:00 PM</td>
<td>3017 Electrical &amp; Computer Eng Bldg</td>
<td>Extra Space</td>
</tr>
</tbody>
</table>
For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.
Common Semiconductors

Fig. 21.4. Room-temperature bandgap energy versus lattice constant of common elemental and binary compound semiconductors.

"Italics" = indirect gap
"Roman" = direct gap
- hexagonal structure
- cubic structure
Contact Information & Website

Professor John M. Dallesasse
2114 Micro and Nanotechnology Laboratory
Office Hours: Tuesdays, 1-2 pm, 2114 MNTL
Office: (217) 333-8416
jdallesa@illinois.edu

John Carlson (TA)
3034 Micro and Nanotechnology Laboratory
Office Hours: Thursdays, 10-11 am, 3034 ECEB
jcarls21@illinois.edu

Website:
https://courses.engr.illinois.edu/ece488/
Course Objectives
Course Objectives

• Develop a working knowledge of compound semiconductor materials and devices
• Provide a foundation for future advanced physical electronics courses
• Provide basic device knowledge to support a career in wireless communications or photonics
• Provide sufficient background such that you can begin to read and understand the literature on compound semiconductor materials and devices
Course Outline
Course Outline

• Review of semiconductor fundamentals
  – Elementary quantum mechanics
  – Atomic bonding and crystal structures
  – Electronic band structures of solids
• Compound semiconductor materials
  – Compound semiconductor crystals
  – Material technologies
• Properties of heterostructures
  – Basic heterostructure properties
  – Electrical properties of heterostructures
  – Optical properties of heterostructures
• Heterostructure devices
  – High-speed electronic devices
  – Semiconductor lasers
  – New device development
Course Description (Detailed)

• Review of quantum, mechanical basics including wave-particle duality, Schroedinger wave equation, one-dimensional free and bounded particles in quantum wells
• Introduction to compound semiconductor crystals, structural and electrical properties, free carrier concentration and Fermi-Dirac integral, III-V alloys
• Phase equilibrium, growth of bulk crystals and phase equilibrium, liquid phase epitaxy, vapor phase epitaxy, metalorganic chemical vapor deposition, molecular beam epitaxy
• Basic heterostructure properties, energy band alignment models, strain effect on the bandgap energies, abrupt p-n heterojunction in equilibrium, heterojunction under bias
• Electronic properties of real quantum wells, potential barrier and tunneling, superlattices and miniband, quantum wells in electric fields, modulation doping and two-dimensional electron gas
• Optical properties of dielectrics, absorption, radiative transitions - Einstein relations, stimulated emission, absorption and emission rates in semiconductors, transitions in degenerated semiconductors, nonradiative recombination processes
• Metal-semiconductor field-effect transistors, pseudomorphomic high-electron mobility transistors, heterojunction bipolar transistors, transfer electron devices, resonant tunneling devices
• Photodetectors, solar cells, light-emitting diodes (LEDs), dielectric waveguide and heterostructure laser theories, quantum well lasers, distributed feedback lasers, vertical cavity surface emitting lasers
Prerequisites

- ECE340 or equivalent basic semiconductor course
- Physics background – Basic modern physics
- Math background – differential equations
# Tentative Schedule [1]

<table>
<thead>
<tr>
<th>AUG 22: Introductions, Objectives, Class Outline, Policies</th>
<th>AUG 24: Motivation, Intro to Quantum Theory</th>
<th>AUG 26: Infinite Square &amp; Triangle Wells</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUG 29: Potential Steps, Coulomb Well (Hydrogen Atom), Atomic Bonding</td>
<td>AUG 31: Crystal Structures, Diffraction</td>
<td>SEP 2: Reciprocal Space, Diffraction Condition</td>
</tr>
<tr>
<td>SEP 5: LABOR DAY NO CLASS</td>
<td>SEP 7: The Brillouin Zone, Band Structures, Density of States</td>
<td>SEP 9: Bloch Theorem, Empty Lattice Model</td>
</tr>
<tr>
<td>SEP 12: Band Gaps</td>
<td>SEP 14: Kronig-Penny Model</td>
<td>SEP 16: Effective Mass, Bloch Oscillations, Band Structure</td>
</tr>
</tbody>
</table>

**Guideline Only: Subject to Change**
Grading and Policies
Grading

<table>
<thead>
<tr>
<th>Grading Category</th>
<th>Percentage of Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homework &amp; Class Participation</td>
<td>30%</td>
</tr>
<tr>
<td>Quizzes (Dates Will be Announced)</td>
<td>10%</td>
</tr>
<tr>
<td>Mid-Term Exam</td>
<td>20%</td>
</tr>
<tr>
<td>Final Exam</td>
<td>40%</td>
</tr>
</tbody>
</table>

Homework:
- Due 1 week after assigned, due in class, no late homework accepted

Quizzes:
- 2 quizzes, dates will be announced ahead of time, 20 minutes

Exam(s):
- Calculator allowed
- 8.5 X 11, hand-written, double-sided formula sheet

Key Points:
- Come to class
- Do your homework
- If you’re having problems attend office hours
Other Comments

• Ask questions if you have them
• Don’t miss quizzes, exams, or homework
• Turn off your cell phones
• No video recording or photography in class
• Include name and NetID on all documents turned in for credit
Class notes (required) can be purchased from the ECE Supply Center
Additional reading materials will be distributed in class or through the course website
Reference for further reading (NOT required):

**Solid state physics:**
- C. Kittel, *Introduction to solid state physics* (any edition), John Wiley

**Semiconductor physics and devices:**
- S.L. Chuang, *Physics of Semiconductor Devices*

**Quantum wells and heterostructures:**

**Compound semiconductor materials:**