\[ n = p \]

To achieve this, \( F_c - F_v = E_g \)

\[ \frac{F_c - E_c}{k_B T} = -\left(\frac{E_v - F_v}{k_B T}\right) \quad \Rightarrow \quad \eta = -\varepsilon \]

Thus, we want \( n = p \) or \( N_c F_y(\eta) = N_v F_y(-\eta) \)

In a GaAs well,

\[ N_c = 2.5 \times 10^{19} \left(\frac{m_e^*}{m_0}\right)^{3/2} = 4.396 \times 10^{17} \text{ cm}^{-3} \]

\[ N_v = 2.5 \times 10^{19} \left(\frac{m_h^*}{m_0}\right)^{3/2} = 9.483 \times 10^{18} \text{ cm}^{-3} \]

\[ F_y(\eta) = 21.87 F_y(-\eta) \]

Looking at the F-D integral table, this happens when \( \eta \approx 2 \)

where \( F_y(2.16) = 12.8237 \)

\[ 21.87 F_y(2.0) = 2.827 \]

\[ n = N_c F_y(\eta) = 2.827 \cdot 4.396 \times 10^{17} \text{ cm}^{-3} = 1.22 \times 10^{18} \text{ cm}^{-3} \]

\[ p = N_v F_y(-\eta) = 0.129 \cdot 9.483 \times 10^{18} \text{ cm}^{-3} = 1.22 \times 10^{18} \text{ cm}^{-3} \]

\[ b. \quad (f_v - f_c) = \frac{1}{1 + \exp\left(-\frac{(m_e^* \cdot E)}{k_B T}\right)} - \frac{1}{1 + \exp\left((E_g + \frac{m_e^* (E - F_c)}{k_B T})\right)} \]

\[ m_e^* = 0.067 m_0 \]

\[ m_h^* = \left(\frac{m_h^*}{m_0}\right)^{3/2} = 0.524 m_0 \]

\[ m_r^* = \left(\frac{1}{m_e^*} + \frac{1}{m_h^*}\right)^{-1} \quad \Rightarrow \quad m_r^* = 0.059 m_0 \quad \text{for a GaAs QW} \]

Plot this function, where \( E = \hbar \omega - E_g \), \( E_g = 1.424 \text{ eV} \)

\[ c. \quad \alpha_o(\hbar \omega) = A^\star \sqrt{\hbar \omega - E_g} \]

From HW 7, \( \alpha_o(\hbar \omega, \text{GaAs}) = 3.122 \times 10^4 \sqrt{\hbar \omega - 1.424} \)

and \( \alpha(\hbar \omega, \text{GaAs}) = \alpha_o(\hbar \omega, \text{GaAs}) \cdot (f_v - f_c) \). Plot.
d) Transparency condition with doping level of \(6 \times 10^{18} \text{cm}^{-3}\)

\[
p = 6 \times 10^{18} \text{cm}^{-3} = 9.483 \times 10^{18} F_{\nu_2}(\eta)
\]

\[
\rightarrow F_{\nu_2}(\eta) = 0.6327 \quad \rightarrow \eta = 0.238 \quad \text{from F-D integral chart,}
\]

\[
F_{\nu_2}(\eta = 0.238) = 0.9214
\]

\[
\rightarrow n = N_c \cdot F_{\nu_2}(\eta = 0.238) = 4.336 \times 10^{17} \cdot 0.9214
\]

\[
= 3.995 \times 10^{17} \text{cm}^{-3}
\]

To have gain, \(n > n_{\text{transparency}}\).

So the injection of electrons, at 1.5x transparency is:

\[
n_{\text{inject}} = 1.5 \cdot n_{\text{transparency}}
\]

\[
= 1.5 \cdot 3.995 \times 10^{17} \text{cm}^{-3}
\]

\[
= 6 \times 10^{17} \text{cm}^{-3}
\]

\[
\rightarrow n_{\text{inject}} = 6 \times 10^{17} \text{cm}^{-3} = 4.336 \times 10^{17} \cdot F_{\nu_2}(\eta)
\]

so \(F_{\nu_2}(\eta) = 1.383 \quad \rightarrow \quad \eta = 0.8031\)

Thus, \(F_c = E_g + k_b T \cdot \eta = 1.424 + 0.025(0.8031) = 1.445\)

\[
F_{\nu} = -\eta k_b T = -0.8031(0.025) = -0.006
\]

Plug into \(\alpha(h\nu, \text{GaAs}) = \alpha_0(h\nu, \text{GaAs}) \cdot (f_{\nu}(F_{\nu}) + f_c(F_c))\)
Gain Curve

Gain coefficient (1/km)

Photon Energy (eV)

\( n = 1.5^n \) (transparency)
\[ E_0 = \left( \frac{5}{2m^*} \right)^{1/2} \left( \frac{9 \pi q^2 N_s}{8 \varepsilon E \varepsilon_r} \right)^{2/3}, \text{ Airy function.} \]

\[ m^*_e = 0.85 m_e^*(\text{GaAs}) + 0.15 m_e^*(\text{InAs}) = 0.85 \left( 0.067m_0 \right) + 0.15 \left( 0.023m_0 \right) = 0.0604 m_0 \]

\[ E(\xi) = 0.85 \varepsilon(\text{GaAs}) + 0.15 \varepsilon(\text{InAs}) = 0.85 \left[ 11.1 \right] + 0.15 \times 15.15 = 13.41 \]

\[ E_0 = \left( \frac{(1.055 \times 10^{-34})^2}{2 \times 0.0604 \times 8.85 \times 10^{-12}} \right)^{1/3} \left( \frac{9 \pi (1.6 \times 10^{-19})^2 \times N_s}{2 \times 13.41 \times 8.85 \times 10^{-12}} \right)^{2/3} \]

\[ = 3.89 \times 10^{-31} \cdot N_s^{2/3} \text{ (Joules)} \approx 2.43 \times 10^{-12} \cdot N_s^{2/3} \text{ (eV)} \]

\[ SE = \frac{\pi \hbar^2 N_s}{m^*} = \frac{\pi (1.055 \times 10^{-34})^2 \times N_s}{0.0604 \times 8.85 \times 10^{-12}} = 6.36 \times 10^{-37} \cdot N_s \text{ (J)} = 3.975 \times 10^{-18} \cdot N_s \text{ (eV)} \]

\[ Ed = 50 \text{ meV (given)} = 0.05 \text{ eV} = 8 \times 10^{-21} \text{ (J)} \]

\[ qV_o = \frac{q^2}{2 \varepsilon N_s^2} = \frac{(1.6 \times 10^{-19})^2 \cdot N_s^2}{2 \times 13.41 \times 8.85 \times 10^{-12} \cdot 2 \times 10^{15} \text{ cm}^{-3} \text{ cm}^3)} = 5.41 \times 10^{-53} N_s^2 \text{ (J)} \]

\[ qV_{sp} = \frac{q^2 W_{sp}}{\varepsilon N_s} = \left\{ \begin{array}{ll}
(1.6 \times 10^{-19})^2 \left( \frac{25 \times 10^{-10}}{13.41 \times 8.85 \times 10^{-12}} \right) N_s = 5.393 \times 10^{-37} N_s \text{ (J)}, \text{ for } 25 \text{Å spacer} \\
(1.6 \times 10^{-19})^2 \left( \frac{100 \times 10^{-10}}{13.41 \times 8.85 \times 10^{-12}} \right) N_s = 2.157 \times 10^{-36} N_s \text{ (J)}, \text{ for } 100 \text{Å spacer}
\end{array} \right. \]

\[ \Delta E_c = E_0 + SE + Ed + qV_o + qV_{sp} \]

\[ = 2.43 \times 10^{-12} N_s^{2/3} + 0.36 \times 10^{-37} N_s + 8 \times 10^{-21} + 5.4 \times 10^{-53} + 5.393 \times 10^{-37} N_s \text{ ( Joules)} \]
Find $\Delta E_c$.

$$E_{v}(Al_{0.2}Ga_{0.8}As) = 0.2 \cdot E_{v}(AlAs) + 0.8 \cdot E_{v}(GaAs) + 3 \cdot (0.2)(0.8) \left[ -\alpha_{v}(AlAs) + \alpha_{v}(GaAs) \right] \frac{\Delta \alpha}{\alpha_0}$$

$$= -6.9254 \text{ eV}$$

$$E_{g}(Al_{0.2}Ga_{0.8}As) = 1.42 + 1.087 \cdot (0.2) + 0.438 \cdot (0.2)^2$$

$$= 1.655 \text{ eV}$$

$$\rightarrow E_{c}(Al_{0.2}Ga_{0.8}As) = E_{g} + E_{v} = -6.9254 \text{ eV} + 1.655 \text{ eV} = -5.27 \text{ eV}$$

$$= -8.43 \cdot 10^{-19} \text{ Joules}$$

$$E_{v}(Ga_{0.85}In_{0.15}As) = 0.85 \cdot E_{v}(GaAs) + 0.15 \cdot E_{v}(InAs) + 3 \cdot (0.85)(0.15) \left[ -\alpha_{v}(GaAs) + \alpha_{v}(InAs) \right] \frac{\Delta \alpha}{\alpha_0}$$

$$= -6.7629 \text{ eV}$$

$$E_{g}(Ga_{0.85}In_{0.15}As) = 0.324 + 0.85(0.7) + 0.4(0.85)^2$$

$$= 1.208 \text{ eV}$$

$$E_{c}(Ga_{0.85}In_{0.15}As) = E_{g} + E_{v} = -6.763 \text{ eV} + 1.208 \text{ eV} = -5.55 \text{ eV}$$

$$= -8.88 \cdot 10^{-19} \text{ Joules}$$

$$\rightarrow \Delta E_c = (-8.43 \cdot 10^{-19} \text{ } - (-8.88 \cdot 10^{-19})) = 4.48 \cdot 10^{-20} \text{ Joules}$$

Combine (1) and (2) and solve for $N_s$:

$$\rightarrow N_s = 1.76 \cdot 10^{12} \text{ cm}^{-2}, \text{ for } W_{sp} = 25 \text{ Å}$$

$$N_s = 1.11 \cdot 10^{12} \text{ cm}^{-2}, \text{ for } W_{sp} = 100 \text{ Å}$$

b. See attached

c. The process is the same as above.
Al$_{x}$Ga$_{1-x}$N, where $x = 0.2$.

To find $n_s$, we know we can use:

$$n_s(x) = \frac{\sigma(x)}{q} - \left(\frac{\varepsilon(x)}{\varepsilon_0}\right)^{\frac{1}{3}}\left[\Phi_b(x) + E_F(x) - \Delta E_c(x)\right]$$

For this, we need the various constants for AlGaN:

$a(x=0) = 3.189 \text{Å} \quad \rightarrow \quad a(x=0.2) = 3.17 \text{Å}$, by interpolation

$\varepsilon(x) = 9.5 + 2.5 \quad \rightarrow \quad \varepsilon(x=0.2) = 9.4$

$q\Phi_b(x) = 1.3x + 0.84 \quad \rightarrow \quad q\Phi_b(x=0.2) = 1.1 \text{eV}$

$E_F(x) = E_0(x) + \frac{\hbar^2}{2m^*(x)} n_s(x)$

$$\rightarrow E_F(x=0.2) = \left(\frac{\hbar^2}{2m^*(x=0.2)}\right)^{\frac{1}{2}} \left(\frac{q\pi n_s(x=0.2)}{8\varepsilon_0 \varepsilon(x=0.2)}\right)^{\frac{2}{3}} + \frac{\pi \hbar^2}{m^*(x=0.2)} \cdot n_s(x=0.2)$$

$$= 1.42 \cdot 10^{-31} n_s(x=0.2)^{\frac{2}{3}} + 1.75 \cdot 10^{-37} n_s(x=0.2)$$

$\Delta E_c(x) = 0.75 \left(x E_C(AlN) + (1-x) E_C(GaN) \right) \rightarrow \Delta E_c(x=0.2) = 0.267 \text{eV}$

$C_{13} = 104 \ \frac{N}{m^2} \quad C_{33} = 398.6 \ \frac{N}{m^2} \quad e_{31}(0.2) = -0.512 \ \frac{C}{m^2} \quad e_{33}(0.2) = 0.876 \ \frac{C}{m^2}$

$P_{sp}(x) = -0.052x - 0.029 \ \frac{C}{m^2} \rightarrow P_{sp}(x=0.2) = -0.0394 \ \frac{C}{m^2}$

$p_{sp}(x=0) = 0.029 \ \frac{C}{m^2}$

$$P_{sp}(x=0.2) = \left(\frac{a(0.2)}{a(0.2)}\right) \left[e_{31}(0.2) - e_{33}(0.2) \cdot \frac{C_{13}}{C_{33}}\right] + P_{sp}(C_2) - P_{sp}(0)$$

$$= 2 \left(\frac{3.189 - 3.17}{3.17}\right) \left[-0.512 - 0.816 \ \frac{10^4}{398.6} \ \frac{C}{m^2} - 0.0394 \ \frac{C}{m^2} - 0.029 \ \frac{C}{m^2}\right]$$

$$= 0.0196 \ \frac{C}{m^2}$$

Solve numerically for $C_{13}$

$$n_s = 9.98 \cdot 10^{12} \text{cm}^{-2}, \quad n_s \approx 1 \cdot 10^{13} \text{cm}^{-2}$$

This verifies original assumption for $x$ to be 0.2.
a. GaSb_{0.6}As_{0.5}/InP

\[ E_v(\text{InP}) = -7.003 \text{ eV} \]
\[ E_g(\text{InP}) = 1.344 \text{ eV} \]
\[ E_c(\text{InP}) = E_v + E_g = -5.66 \text{ eV} \]

\[ E_v(\text{GaAs}) = -6.807 \text{ eV} \quad \text{from model-solid} \]
\[ E_v(\text{GaSb}) = -5.97 \text{ eV} \]
\[ E_v(\text{GaSb}_{0.6}\text{As}_{0.5}) = 0.5(-6.8) + 0.5(-5.97) = -6.39 \text{ eV} \]

\[ E_g(\text{GaSb}_{0.6}\text{As}_{0.5}) = 1.43 - 0.5(1.9) + 1.2(0.5)^2 \text{ eV} \quad \text{from Table 4-8} \]
\[ = 0.78 \text{ eV} \]
\[ E_c(\text{GaSb}_{0.6}\text{As}_{0.5}) = E_v + E_g = -5.6 \text{ eV} \]

\[ \Delta E_v = 0.66 \text{ eV} \quad \Delta E_c = 0.613 \text{ eV} \]

b. GaIn_{0.3}As/InP vs. GaSb_{0.6}As_{0.5}/InP

\[ \Delta E_v = 0.36 \text{ eV} \]
\[ \Delta E_c = 0.252 \text{ eV} \]

\[ \frac{1}{\text{Ratio}} = \frac{\beta_{\text{max}}(\text{GaIn}_{0.3}\text{As})}{\beta_{\text{max}}(\text{GaSb}_{0.6}\text{As}_{0.5})} = \frac{N_e P_{np} 2 \hbar \pi}{P_{np} N_e 2 \hbar \pi} \cdot \frac{e^{\Delta E_g/k_B T}}{e^{\Delta E_g/k_B T}} \]
\[ = e^\frac{(E_g(\text{GaIn}_{0.3}\text{As}) - E_g(\text{InP})) - (E_g(\text{GaSb}_{0.6}\text{As}_{0.5}) - E_g(\text{InP}))}{k_B T} \]
\[ = e^{\frac{1}{k_B T} \cdot (0.7199 - 0.78)} = 0.09 \]

\[ \rightarrow \text{Ratio} = 11.06 \]
V. Strained InGaAs Laser

From the paper cited by Groves, Wolpope, Missaggia (1992):

\[ R = 0.32\% \quad \text{GaAs} \quad \text{or} \quad \text{InGaAsP cladding} \quad L = 0.98 \mu m \]

2b) \( \alpha_i, \eta_i \):

Since \( \eta_d = \frac{\alpha_m}{\alpha_i + \alpha_m} \), where \( \alpha_m \) is mirror loss, \( \alpha_i \) is intrinsic material loss

and \( \eta_i \) is the intrinsic quantum efficiency

\( \eta_i \) is found when length is so large that \( \alpha_i \) losses outdo \( \alpha_m \) losses.

So when \( \eta_d \to \frac{1}{\alpha_i(L)} \), \( \eta_d = \eta_i \).

Thus \( \eta_i \) is read off the attached plots as the \( y \)-intercept.

\( \eta_i \left( \text{InGaAsP cladding} \right) = 0.35 - 0.39 \)

\( \eta_i \left( \text{GaAs cladding} \right) = 0.22 - 0.25 \)

To find \( \alpha_i \), we use \( \frac{1}{\eta_d} = \frac{1}{\eta_i} \left( 1 + \alpha_i \frac{L}{\ln(R)} \right) = \frac{1}{\eta_i} \left( 1 - \alpha_i \frac{L}{\ln(0.32)} \right) \)

\( \alpha_i \left( \text{InGaAsP cladding} \right) = 1.89 \cdot 10^2 \text{ m}^{-1} = 1.89 \text{ cm}^{-1} \)

\( \alpha_i \left( \text{GaAs cladding} \right) = 1.02 \cdot 10^3 \text{ m}^{-1} = 10.2 \text{ cm}^{-1} \)

c) Transparency is the point when \( \frac{F_c - F_V}{k_B T} = E_g \), so any \( J \) above this acts as a threshold for the QW. Thus, the \( y \)-intercept on the \( J \)th vs. \( \frac{1}{L} \) plot is \( J_{\text{transparency}} \).

\( J_{\text{transparency}} (\text{InGaAsP cladding}) = 40 \frac{A}{cm^2} \)

\( J_{\text{transparency}} (\text{GaAs}) = 50 \frac{A}{cm^2} \)