

Reading Assignment:

FPE, Sections 6.3, 6.7.1–6.7.6.

Problems:

1. Consider the transfer function

$$H(s) = \frac{1}{s + a},$$

where $a > 0$. Prove that the Nyquist plot of H is a circle of radius $\frac{1}{2a}$ centered at the point $\left(\frac{1}{2a}, 0\right)$.

2. For the two plant transfer functions given below, use the Nyquist stability criterion to determine all values of the feedback gain K that stabilize the closed-loop system.

$$(a) G(s) = \frac{1}{(s + 2)(s + 5)} \quad (b) G(s) = \frac{1}{(s + 2)(s^2 + 2s + 5)}$$

Instructions: To draw the Nyquist plot, use the Bode plots of $G(s)$. Explain all steps in arriving at the Nyquist plot. Start with hand-sketched Bode plots. You can then generate more accurate Bode plots with MATLAB to get exact numerical values if necessary. *The use of MATLAB for drawing the Nyquist plot is not allowed* (except to check your work at the end). It is also recommended that you check your results with Routh-Hurwitz stability criterion.

3. For the two transfer functions and gain values given below, use the Nyquist plot to find the gain and the phase margins:

$$(a) G(s) = \frac{1}{(s - 1)(s + 2)(s + 4)}, \quad K = 10;$$

$$(b) G(s) = \frac{1}{(s + 1)^3}, \quad K = 3.$$

Instructions: Follow the same instructions as in the previous problem. *You may (and should) use MATLAB for drawing a more accurate Nyquist plot to find the GM and the PM, but you must explain all your work on the hand-sketched Nyquist plot.*