ECE 486 **Assignment # 6**

Issued: March 24 Due: March 31, 2022

Reading Assignment:

FPE, Sections 6.1, 6.2, 6.4–6.6.

Problems:

(you can use MATLAB in Problems 3 and 4, but you must explain all steps and justify all answers)

1. For each of the transfer functions given below, draw the Bode plots (both magnitude and phase) *by hand*, using the techniques discussed in class. Explain all steps in your drawing procedures. Note that the transfer functions are not given in Bode form.

   $$\text{(a) } KG(s) = \frac{s+8}{s(s+4)} \quad \text{(b) } KG(s) = \frac{8s}{s^2+0.2s+4} \quad \text{(c) } KG(s) = \frac{s^2+0.2s+1}{s(s+0.2)(s+6)}$$

   After you're done, check your results using MATLAB. (Note that the `bode` command in MATLAB plots magnitude in decibels.) Turn in both the hand sketches and the MATLAB plots.

2. Show that for the transfer function $KG(s) = \dfrac{\omega_n^2}{s^2+2\zeta\omega_n s}$, the phase margin is independent of ω_n and is given by

   $$\text{PM} = \tan^{-1}\left(\frac{2\zeta}{\sqrt{\sqrt{4\zeta^4+1}-2\zeta^2}}\right).$$

3. Consider the transfer function $G(s) = \dfrac{1}{s(s^2+4s+8)}$.

   (i) Derive the value of K for which the closed-loop characteristic equation $1+KG(s)$ has roots on the $j\omega$-axis.

   (ii) For this value of K, make the Bode plot of $KG(s)$ using MATLAB and explain how you can confirm the presence of $j\omega$-axis closed-loop poles using this plot.

   (iii) Compute the gain and phase margins for $K = 12$ using the corresponding Bode plot.

   (iv) Determine the gain K that gives the phase margin of $60°$.

   (v) Plot the step responses of the closed-loop systems for $K = 12$ and the K you found in part (iv). Which system has better damping (smaller overshoot)? Why?

4. Consider the system

   $$G(s) = \frac{10}{\left(\frac{s}{0.2}+1\right)\left(\frac{s}{0.5}+1\right)}.$$

   We wish to design a lead/lag controller that provides bandwidth of at least 2, PM of at least $60°$, and steady-sate tracking of constant references within 1%.

(i) For the controller

$$KD(s) = 4\frac{\frac{s}{0.8}+1}{\frac{s}{5}+1} \cdot \frac{s+0.05}{s+0.02}$$

derived in class, compute the PM, bandwidth, and steady-state tracking error to verify whether the specs are met.

(ii) Suppose that in addition to the above specs, the bandwidth cannot exceed 6. Modify the design to incorporate this new spec, and verify that it indeed works.