Problem 1. Consider the single-input, single-output transfer function:

\[ Y(s) = \frac{s + 1}{s^2 + 2s + 2} U(s) \]

(a) Find a second-order state-space model that represents this transfer function.

(b) For this state-space model, calculate a state-feedback controller \( u = -Kx + r \) that places the closed-loop poles at \(-4\) and \(-25\).

(c) Construct a stable observer to estimate \( x \) based on the known inputs \( u \) and observations \( y \). You may use MATLAB for this part.

(d) With the controller and observer from the previous problems in place, calculate \( k_r \) such that \( u = -K\dot{x} + k_rr \) yields a closed-loop system \( Y/R \) with unity gain. You may use MATLAB.

(e) Plot the step response using MATLAB.

Problem 2. Consider the single-input, single-output transfer function:

\[ G_p(s) = \frac{1 - s/2}{1 + s/2} \frac{1}{s^2} \]

(a) Find a third-order state-space model that represents this transfer function.

(b) For this state-space model, calculate a state-feedback controller \( u = -Kx + r \) that places the closed-loop poles at \(-4\), \(-13\), and \(-25\). You may use MATLAB to calculate this controller, but not to find the state-space model.

(c) Construct a stable observer, and put this together to form a compensator of the form \( U = -G_cY + G_rR \). You may use MATLAB.

(d) Calculate the Nyquist plot of \( G_cG_p \). You may use MATLAB to do so. Is the system stable? If so, calculate the gain and phase margins.