ECE 486 (Control Systems) – Homework 3

Due: Feb. 13

Problem 1. Find the transfer function for the block diagram in Figure 1. The answer should be given in terms of the block's transfer functions, i.e. K, G, H, P.

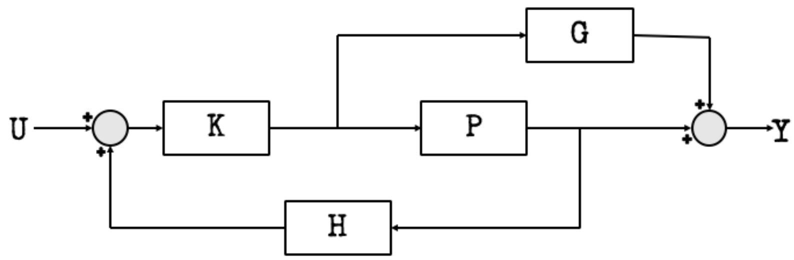

Figure 1: The block diagram for Problem 1.

Problem 2. Consider Figure 2. All state variables, inputs, and outputs are scalars.

i) Find the transfer function from the input R to the output Y. Your answer should be the ratio of two polynomials in s, with coefficients expressed in terms of a, b, c, k, l, m.

ii) Write down the conditions that must be satisfied by a, b, c, k, l, m for this transfer function to be stable, i.e. for all poles of the transfer function to have negative real parts.

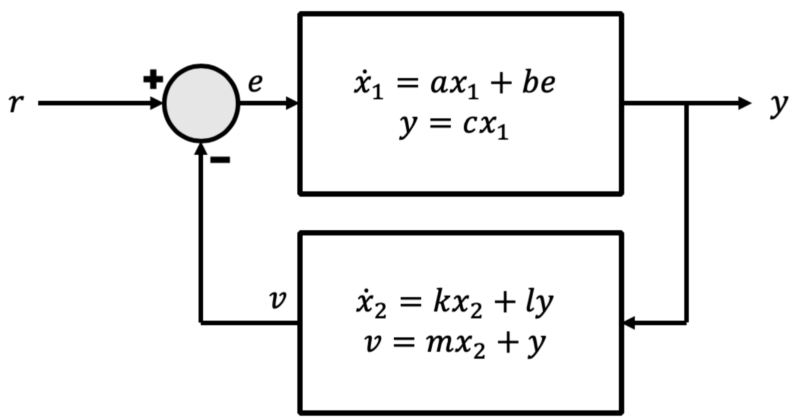

Figure 2: The block diagram for Problem 2.

Problem 3. Recall the dynamics for the mass-spring system from lecture, as depicted in Figure 3. The dynamics are:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\rho}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Here, k is the spring constant and ρ is the friction coefficient.

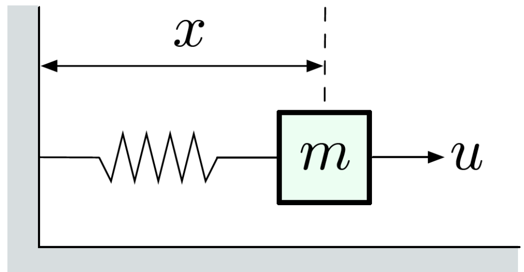

Figure 3: The mass-spring system discussed in lecture.

i) Find the transfer function of this system.

ii) Suppose that the C matrix is replaced, such that:

$$y = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Recalculate the transfer function with this sensor model. Which values of c_1 and c_2 guarantee the new system has the form of the prototypical 2nd-order response discussed in class? Write ω_n and ζ in terms of k, ρ, m.

Determine the steady-state response of this new system to the sinusoidal external force $u(t) = \sin(\omega t)$, where ω is a chosen constant.

Sketch or print plots of the magnitude and phase shift of the steady state response as functions of the input frequency ω. What happens when the input frequency equals the system's natural frequency, i.e. $\omega = \omega_n$? (This phenomena is known as *resonance*.)

Problem 4. Consider the transfer function:

$$H(S) = \frac{25}{s^2 + 8s + 25}$$

i) Suppose you are given the following time-domain specs: rise time $t_r \leq 0.6$ and settling time $t_s \leq 1.6$. (Here we're considering settling time to within 5% of the steady-state value.) Plot the admissible pole locations in the s-plane corresponding to these two specs. Does this system satisfy these specs?

ii) Repeat the previous problem for the specs: rise time $t_r \leq 0.6$, settling time $t_s \leq 1.6$, and magnitude $M_p \leq 1/e^2$. Plot the admissible pole locations; does this system satisfy these specs?

iii) Repeat the previous problem for the specs: rise time $t_r \leq 0.6$, settling time $t_s \leq 1.6$, and peak time $t_p \leq 1$. Plot the admissible pole locations; does this system satisfy these specs?