

# ECE 486 (Control Systems) – Homework 1

Due: Jan. 30

## Block diagrams and modeling

**Problem 1.** Pictured in Figure 1 is a sketch of Drebbel's incubator. The alcohol will expand or contract depending on the temperature of the water. This, in turn, adjusts the damper.

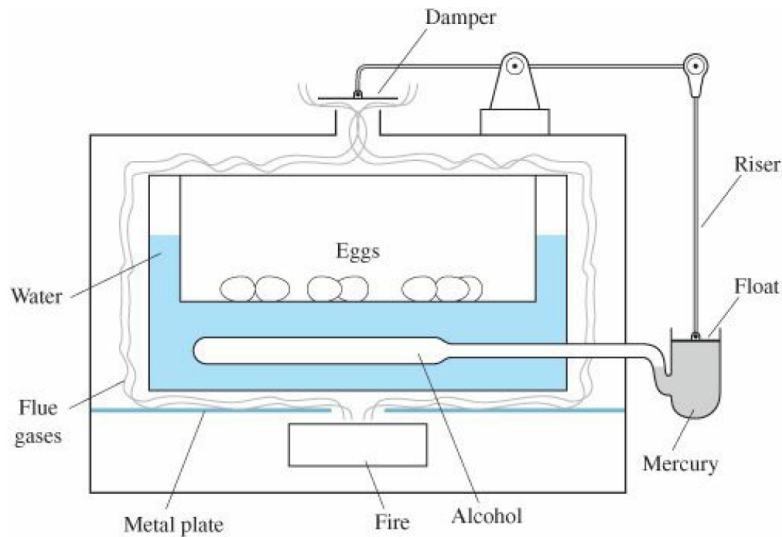


Figure 1: Drebbel's incubator was an early feedback control system for incubating chicken eggs. It was invented around 1620.

Draw a component block diagram for Drebbel's incubator. Identify the system output, plant, sensors, and controller. Describe the process for each.

## Linear algebra review

**Problem 2.** Calculate the characteristic polynomial and eigenvalues for each of the matrices below.

As a reminder, the characteristic polynomial of a matrix  $A$  is given by  $p_A(s) = \det(sI - A)$ , where  $I$  is the identity matrix with dimensions matching  $A$ , and  $sI$  is the identity matrix  $I$  multiplied by a scalar  $s$ .

i)  $A = \begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}$

ii)  $A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$

iii)  $A = \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix}$

## Complex numbers review

**Problem 3.** Calculate the magnitude and phase of the following complex numbers.

i)  $x = 3 + 4j$

ii)  $x = 21 - 20j$

iii)  $x = a/b$ , where the magnitudes  $|a|$  and  $|b|$  are given, as well as the phases  $\angle a$  and  $\angle b$ .

## Putting ODEs in state-space form

**Problem 4.** Take the following ordinary differential equations (ODEs) and write them in state-space form, i.e. equations of the form  $\dot{x} = Ax + Bu$ .

We use the notation where  $x^{(n)}$  denotes the  $n$ th derivative of  $x(t)$ . Be careful with minus signs!

i)  $x^{(5)} - x^{(4)} + 3x^{(2)} = 16x^{(1)} + 12x - 2u$

ii)  $x^{(4)} + a_3x^{(3)} + a_2x^{(2)} + a_1x^{(1)} + a_0x = u$ . Here, each  $a_i$  is a known constant.

## Deriving dynamics from a linear circuit

**Problem 5.** Derive a state space model of the form  $\dot{x} = Ax + Bu$  that models the dynamics of the RLC circuit in Figure 2. Use  $V_S$  as the input. You may choose what your states are, but explicitly declare your choice.

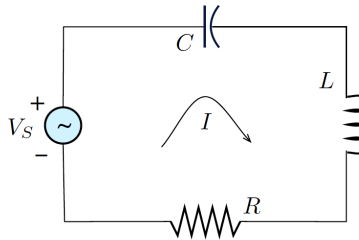


Figure 2: A typical RLC circuit.

## Nonlinear dynamics

**Problem 6.** Consider the following second-order differential equation:

$$\frac{d^2y}{dt^2} - (1 - y^2)\frac{dy}{dt} + y = 0$$

i) Write the dynamics as a non-linear state-space equation.

**Remark:** State-space models that do not have an input, i.e. are of the form  $\dot{x} = Ax$ , are ‘autonomous’, since they evolve on their own<sup>1</sup>.

ii) Identify all equilibria of the system, i.e. points  $x$  such that  $\dot{x} = 0$ . You must both find these equilibria and argue that there are no others.

iii) For each equilibrium point, linearize your dynamics about said equilibrium point, and give the linearized dynamics in state-space form, i.e.  $\dot{x} = Ax$ .

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<sup>1</sup>Care must be taken with this terminology, as an ‘autonomous’ system means something different to mathematicians. In mathematics, a differential equation  $\frac{dy}{dx} = f(y)$  is autonomous because the right-hand side does not depend on  $x$ .