ECE 486 (Control Systems) – Homework 7

Problem 1. Let $a > 0$ and consider:

$$H(s) = \frac{1}{s+a}$$

Prove that the Nyquist plot of $H(s)$ is a circle of radius $\frac{1}{2a}$ centered at $\left(\frac{1}{2a}, 0\right)$.

Problem 2. Consider the following:

$$G(s) = \frac{1}{(s+2)(s+5)}$$

Suppose this is in unity feedback with a constant gain controller K. In other words, we have a negative feedback loop where the forward gain is $KG(s)$ and the loop gain is also $KG(s)$.

i) Use the Routh-Hurwitz stability criterion to determine what values of K stabilize the closed-loop system.

ii) Sketch the Bode plots of $G(s)$ by hand.

iii) Verify your Bode plots using MATLAB.

iv) Using the Bode plot, sketch the Nyquist plot by hand. You should primarily use your sketch of the Bode plot, but you may use MATLAB to calculate exact numerical values as needed.

v) Using the Nyquist plot, determine what values of K stabilize the closed-loop system. Does this match your answer from the Routh-Hurwitz criterion?

Note: In this problem, I have explicitly provided a step-by-step guide on how to use Nyquist plots to determine stability of a system. You are expected to be able to understand these principles and do this on your own, e.g. a problem on an exam may be "Use the Nyquist plots to determine the values of K which stabilize the closed-loop system." with no further guidance provided.

Problem 3. Repeat the previous problem for:

$$G(s) = \frac{1}{(s+2)(s^2+2s+5)}$$

Problem 4. Consider:

$$G(s) = \frac{1}{(s-1)(s+2)(s+4)} \qquad K = 10$$

i) Sketch the Bode plot of $KG(s)$ by hand.

ii) Use the Bode plot to sketch the Nyquist plot.

iii) Use MATLAB to draw an exact Nyquist plot.

iv) Use this Nyquist plot to calculate the gain margin and phase margin.

Problem 5. Repeat the previous problem for:

$$G(s) = \frac{1}{(s+1)^3} \qquad K = 3$$