ECE 486 (Control Systems) – Homework 6

Problem 1. For $\omega = 1, 2, 5, 10, 20, 50, 100$, calculate the magnitude and phase of $G(s)$ by hand, where:

$$G(s) = \frac{1}{s + 10}$$

Sketch the Bode plot for $G(s)$ and compare the calculated points with the Bode plot.

Problem 2. Sketch the asymptotes of the Bode plot magnitude and phase for each of the following. Verify your results in MATLAB; turn in both your hand sketches and MATLAB results.

i) $L(s) = \frac{100}{s(0.1s+1)(0.5s+1)}$

ii) $L(s) = \frac{10(s+4)}{s(s+1)(s+200)}$

iii) $L(s) = \frac{s+2}{s(s+10)(s^2+2s+2)}$

Problem 3. Suppose we have the open-loop transfer function $G(s) = \frac{K}{s(s+20)(s+85)}$, and we put it through unity feedback, i.e. the closed loop transfer function is $\frac{G(s)}{1+G(s)}$.

i) Set the gain K so that the magnitude is 1 (0 dB) at $\omega = 1$. What value of K achieves this?

ii) We wish to achieve 15% overshoot in the transient response for a step input. What phase margin is required to achieve this?

iii) What frequency on the Bode phase diagram yields this phase margin?

iv) Find the adjusted gain necessary to produce the required phase margin.

v) Using MATLAB, plot the step response of the compensated system. Did you meet the design specs?

Problem 4. For a transfer function $G(s) = \frac{K}{s(s+20)(s+85)}$ in unity feedback, design a lag compensator to reduce the steady-state error for a ramp input by a factor of 10 while maintaining a 15% overshoot in the transient response for a step input.

i) Determine the steady state error of the gain-adjusted system designed in the previous problem.

ii) Find a gain K to satisfy the steady-state specification and plot the Bode plot in MATLAB for this gain.

iii) Determine the phase margin to achieve the desired overshoot. Find the frequency where the phase margin is 10 degrees greater than this phase margin.

iv) Design a lag compensator to achieve a gain of 1 (0 dB) at this frequency. In particular, set a high-frequency asymptote so that the compensated system will have a gain of 1 at this frequency, set the low-frequency asymptote to be at 1 (0 dB), and then connect the two with a -1/decade line (-20 dB/decade line). Find the lag compensator that achieves this.

v) Why do we increase the phase margin above the desired margin when designing a lag compensator?

vi) Did you meet the design specifications? Include relevant plots.