ECE 486 (Control Systems) – Homework 5

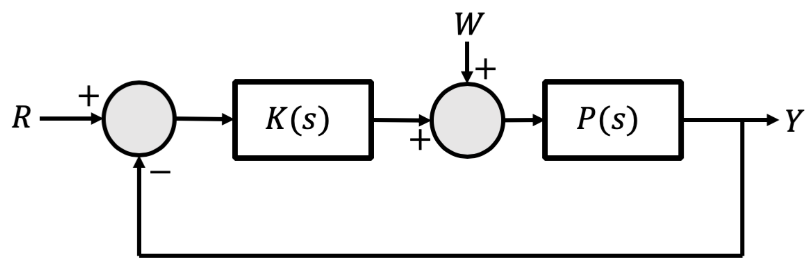

Figure 1: A diagram of a unity feedback system with input disturbances.

Problem 1. Recall that a closed-loop system is of type n with respect to the reference input if the forward-loop transfer function $K(s)P(s)$ has a pole of order n at the origin, or, equivalently, if:

$$\lim_{s\to 0}\left[s^n K(s)P(s)\right] = \text{const} \neq 0$$

The above equation states that the the limit exists, the limit is finite, and the limit is non-zero.

Assuming the closed-loop system is stable, that means that n is the lowest degree of a polynomial that cannot be tracked in feedback with zero steady-state error. This is what is commonly referred to as system type, and can be thought of as system type with respect to the reference input. The convention is that 'system type' typically refers to system type in this sense.

However, we can think of other formulations of system type. In this problem, we'll consider a generalization that considers system type with respect to disturbances. Consider the unity feedback configuration with an additive input disturbance W in Figure 1.

i) Let $T_{r\to y}(s)$ denote the transfer function from R to Y.

   Remark: When our system has multiple inputs and one output, we ignore the other inputs when calculating the transfer function from one input to the output.

   Show that the system type with respect to reference inputs is n whenever:

$$\lim_{s\to 0}\frac{1-T_{r\to y}(s)}{s^n} = \text{const} \neq 0$$

ii) Let $T_{w\to y}(s)$ denote the transfer function from W to Y.

   We say the system has type k with respect to disturbance inputs if:

$$\lim_{s\to 0}\frac{T_{w\to y}(s)}{s^k} = \text{const} \neq 0$$

   Show that if $T_{w\to y}(s)$ has a zero of order k at the origin, then the system has type k with respect to disturbance inputs.

   Remark: If $T_{w\to y}(s)$ has a zero of order k at the origin, then we can write it as $T_{w\to y}(s) = s^k\frac{A(s)}{B(s)}$ where A and B are polynomials with real coefficients such that $A(0)\neq 0$ and $B(0)\neq 0$.

iii) Show that the system of type k with respect to disturbance inputs can achieve perfect steady-state disturbance rejection with polynomial disturbances with degree $m < k$, but not when $m \geq k$.

iv) Finally, consider the plant:

$$P(s) = \frac{1}{s^2 + 1}$$

Determine the system type with respect to disturbances under P-control ($K(s) = K_P$), PD-control ($K(s) = K_P + K_D s$), and PID-control ($K(s) = K_P + K_D s + \frac{K_I}{s}$).

Problem 2. Sketch the root loci for the following $L(s)$ by hand. (Recall that the root locus plots how the solutions of $1 + KL(s) = 0$ vary as K goes from 0 to $+\infty$.

i) $L(s) = \frac{1}{s^2+2s+10}$

ii) $L(s) = \frac{s-2}{s^2+2s+10}$

iii) $L(s) = \frac{(s+1)(s+2)}{s(s^2+4)(s^2+5)}$

iv) $L(s) = \frac{s+2}{s^5+1}$

Problem 3. Consider the feedback system in Figure 2.

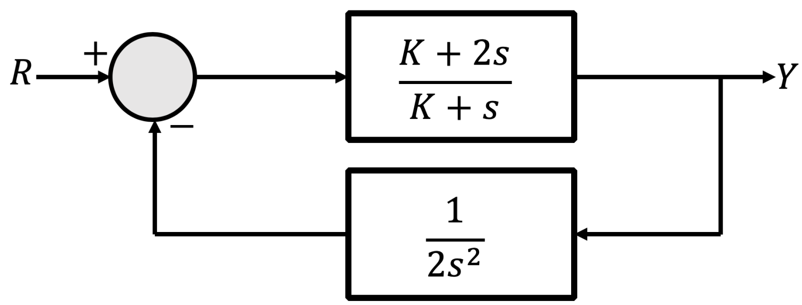

Figure 2: A feedback system.

Determine the transfer function for $Y(s)/R(s)$, and write the characteristic equation in terms of K. In other words, find the polynomials $a(s)$ and $b(s)$ such that the closed loop poles are the values of s that satisfy to $a(s)+Kb(s) = 0$.

You do **not** have to do this for this problem, but you should understand how to draw a root locus for where the closed loop pole locations as K goes from 0 to $+\infty$. (This shows that the root locus methods apply to more systems than the constant-gain unity-feedback setting discussed in class.)

Problem 4. Suppose we have a constant-gain unity-feedback control system in the style of Evans, as discussed in lecture. In other words, the closed loop transfer function is given by:

$$\frac{KL(s)}{1 + KL(s)}$$

i) If $L(s)$ has 3 LHP poles and 1 LHP zero, is the closed-loop system stable for very large values of $K > 0$?

ii) If $L(s)$ has 2 LHP poles, 1 RHP poles, and 3 LHP zeros, is the closed-loop system stable for very large values of $K > 0$?

iii) If $L(s)$ has 5 LHP poles, 4 LHP zeros, and 1 RHP zeroes, is the closed-loop system stable for very large values of $K > 0$?

Give detailed justification for each answer.