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- Today's topic: linear systems and their dynamic response

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Reading: FPE, Section 3.1, Appendix A.

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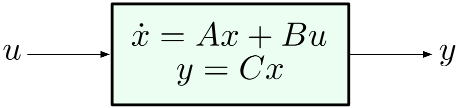
Impulse Response

(Review from ECE 210)



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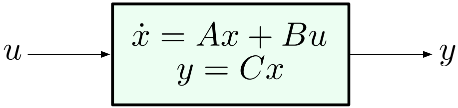

Unit impulse (or Dirac's δ-function):

Impulse Response

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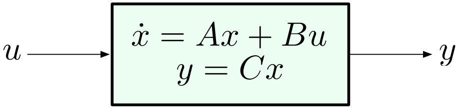

Unit impulse (or Dirac's δ-function):

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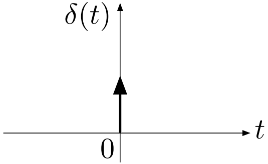
2. $\displaystyle\int_{-a}^{a} \delta(t)\mathrm{d}t = 1$ for all $a > 0$



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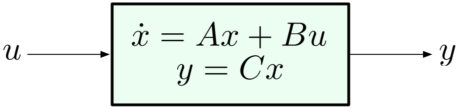

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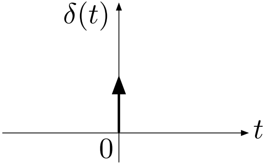

It is useful to think of $\delta(t)$ as a limit of impulses of unit area:

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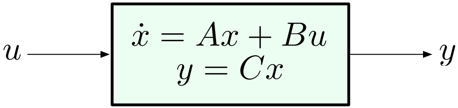

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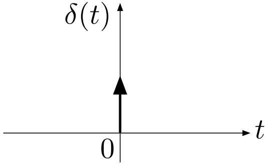

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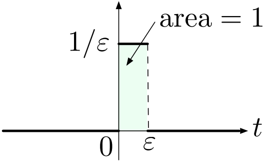
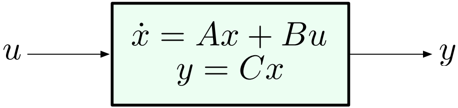


as $\varepsilon \to 0$, the impulse gets taller ($1/\varepsilon \to +\infty$), but the area under its graph remains at 1

Impulse Response



zero initial condition: $x(0) = 0$

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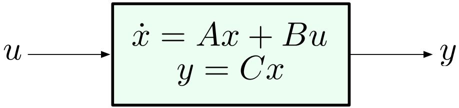

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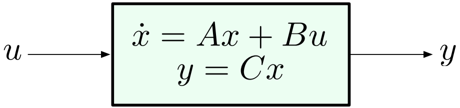

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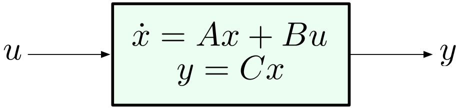

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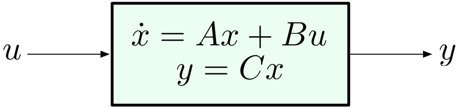

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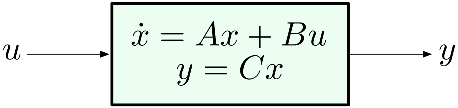

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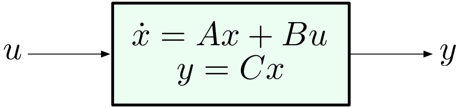
The function h is the impulse response of the system.

Impulse Response



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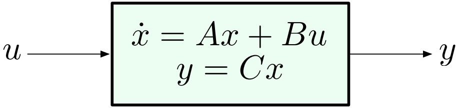

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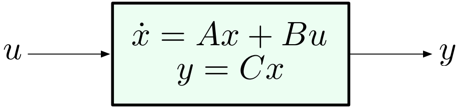

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Questions to consider:

1. If we know h, how can we find the system's response to other (arbitrary) inputs?
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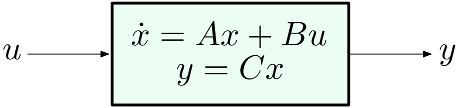

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We will start with Question 1.

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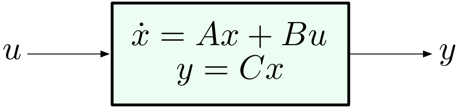

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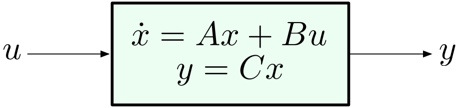

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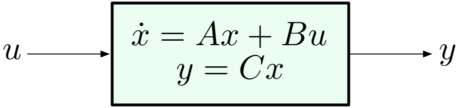

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— any *reasonably regular* function can be represented as an integral of impulses!!

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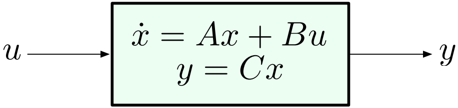

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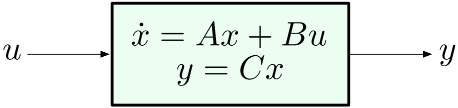

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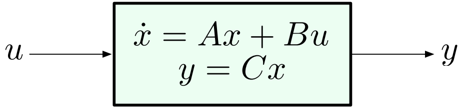

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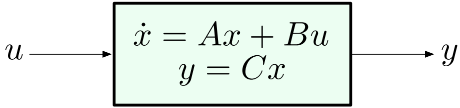

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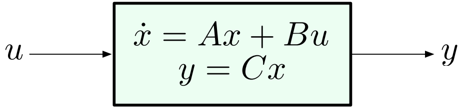

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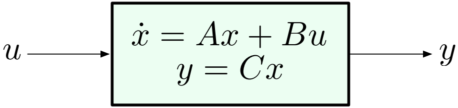

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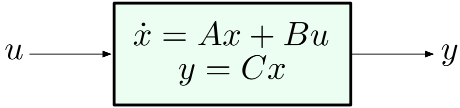

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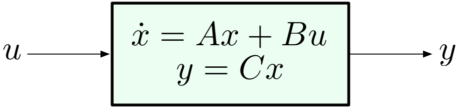

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— the integral that defines $y(t)$ is a convolution of u and h.

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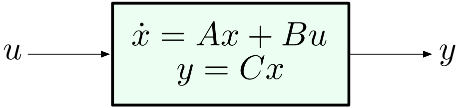

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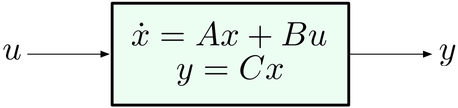

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Conclusion so far: for zero initial conditions, the output is the convolution of the input with the system impulse response:

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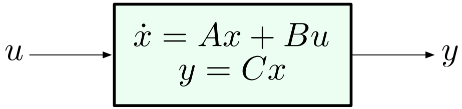

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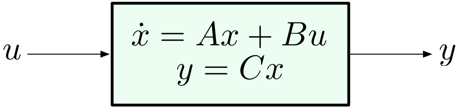

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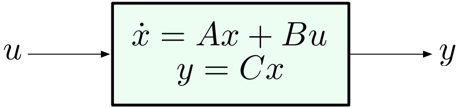

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Laplace Transforms and the Transfer Function

Reminder: the *two-sided* Laplace transform of a function $f(t)$ is

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time domain	frequency domain
$u(t)$	$U(s)$
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Laplace Transforms and the Transfer Function

Reminder: the *two-sided* Laplace transform of a function $f(t)$ is

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The Laplace transform of the impulse response

$$H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} \mathrm{d}\tau,$$

is called the transfer function of the system.

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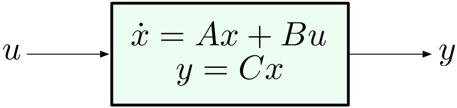

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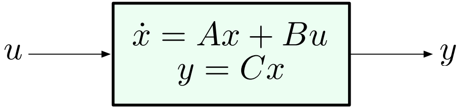

In this case, we have an exact formula:

$$H(s) = C(Is - A)^{-1}B \qquad \text{(matrix inversion)}$$

$$h(t) = Ce^{At}B, \; t \geq 0^{-} \qquad \text{(matrix exponential)}$$

— will not encounter this until much later in the semester.

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  – so, $u(t) = e^{st}$ is multiplied by $H(s)$ to give the output.

Example

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$$u(t) = e^{st} \qquad \text{(always assume } u(t) = 0 \text{ for } t < 0\text{)}$$

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$$H(s) = \frac{1}{s+a} \qquad \Longrightarrow \qquad y(t) = \frac{e^{st}}{s+a}$$

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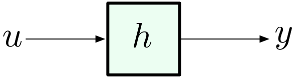
Now we can fund the impulse response $h(t)$ by taking the inverse Laplace transform — from tables,

$$h(t) = \begin{cases} e^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Determining the Impulse Response



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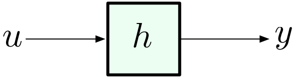

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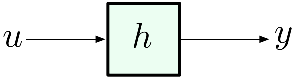

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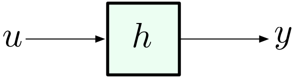

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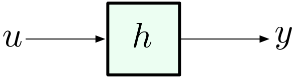

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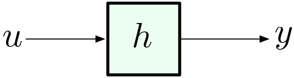

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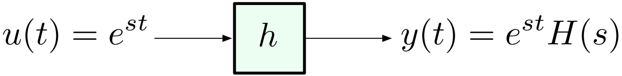

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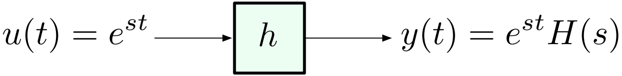

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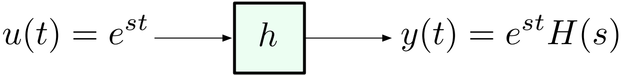

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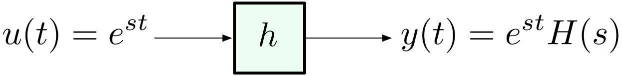

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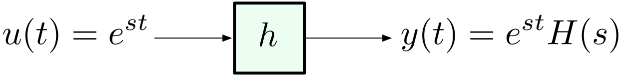

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This is possible if we allow s to take on *complex values*.

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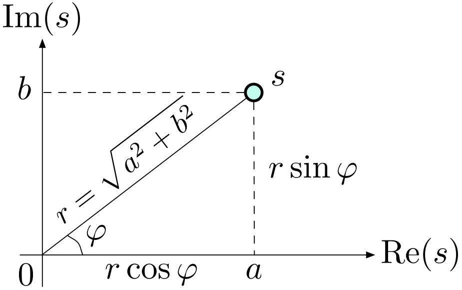

Polar form:

$$s = re^{j\varphi}$$

$$r = |s| = \sqrt{a^2 + b^2}$$

(magnitude)

$$\varphi = \angle s = \tan^{-1}\left(\frac{b}{a}\right)$$

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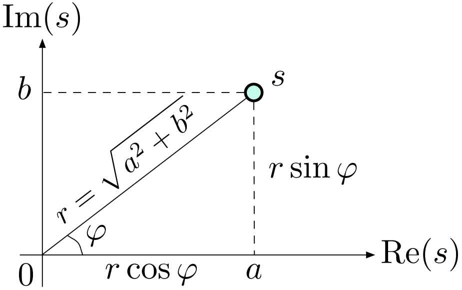

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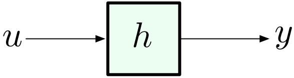
Euler's formula: $e^{j\varphi} = \cos\varphi + j\sin\varphi$

Frequency Response



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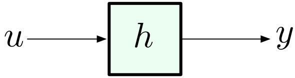
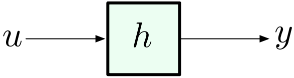


$u(t) = A\cos(\omega t)$ $\quad$ A – amplitude; ω – (angular) frequency, rad/s

From Euler's formula:

$$A\cos(\omega t) = \frac{A}{2}\left(e^{j\omega t} + e^{-j\omega t}\right)$$

Frequency Response



$u(t) = A\cos(\omega t)$ $\quad$ A – amplitude; ω – (angular) frequency, rad/s

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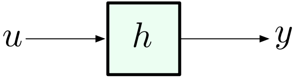
Frequency Response



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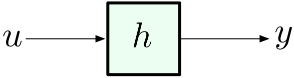

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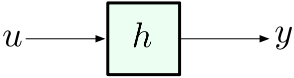

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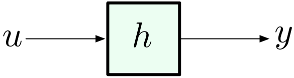

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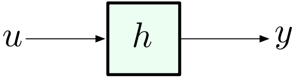
Frequency Response



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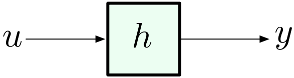
(recall that $h(\tau)$ is real-valued)

Frequency Response



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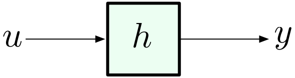
Frequency Response



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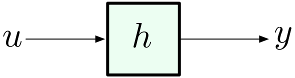

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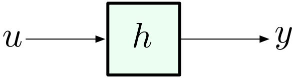

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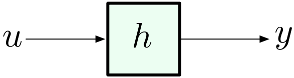

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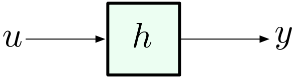

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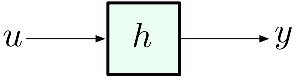
The (steady-state) response to a cosine signal with amplitude A and frequency ω is still a cosine signal with amplitude $AM(\omega)$, same frequency ω, and phase shift $\varphi(\omega)$

Frequency Response



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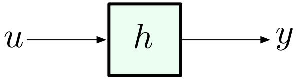
Frequency Response



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Still an incomplete picture:

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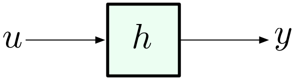

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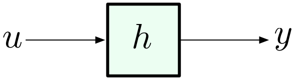

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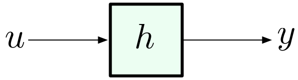

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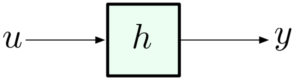

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$$\text{total response} = \begin{matrix}\text{transient response}\\ \text{(depends on I.C.)}\end{matrix} + \begin{matrix}\text{steady-state response}\\ \text{(independent of I.C.)}\end{matrix}$$

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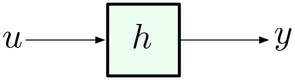

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— need more on Laplace transforms