Solution 1

(i) 
\[
\begin{array}{cccc}
  s^4 & 1 & 15 & 1 \\
  s^3 & 10 & 20 & \\
  s^2 & 13 & 1 & \\
  s^1 & 250 & 13 & \\
  s^0 & 1 & \\
\end{array}
\]
No change of sign in the first column ⇒ No RHP roots.

(ii) There are negative coefficients ⇒ RHP roots exist.

(iii) There are negative coefficients ⇒ RHP roots exist.

(iv) 
\[
\begin{array}{cccc}
  s^4 & 1 & 12 & 1 \\
  s^3 & 10 & 20 & \\
  s^2 & 10 & 1 & \\
  s^1 & 19 & & \\
  s^0 & 1 & & \\
\end{array}
\]
No change of sign in the first column ⇒ No RHP roots.

Solution 2

The closed loop transfer function is:
\[
G_{cl} = \frac{KG}{1 + KG} = \frac{\frac{K}{s^3 + 3s^2 + s + 1}}{1 + \frac{s^3 + 3s^2 + s + 1}{s^3 + 3s^2 + s + 1}} = \frac{K}{s^3 + 3s^2 + s + (K + 1)}
\]

Construct the Routh-Hurwitz array:
\[
\begin{array}{cccc}
  s^3 & 1 & 1 & \\
  s^2 & 3 & (K + 1) & \\
  s^1 & 3 - (K + 1) & & \\
  s^0 & K + 1 & & \\
\end{array}
\]
Hence for the system to be stable, we need:

\[
\frac{3-(K+1)}{K+1} > 0 \implies -1 < K < 2
\]

In addition, the system is unstable when \( K \geq 2 \)

**Solution 3**

(i) Constant reference, say unit step: \( R(s) = \frac{1}{s} \). Assume there is no disturbance, i.e., \( W = 0 \). Then

\[
Y = KGR = \frac{K}{s(s+p)}
\]

Using Final Value Theorem,

\[
y(\infty) = r(\infty) = 1 \implies 1 = \lim_{s \to 0} Ys = \lim_{s \to 0} \frac{K}{s+p} = \frac{K}{p} \implies K = p
\]

(2) Constant disturbance, say unit step: \( W(s) = \frac{1}{s} \). Assume there is no reference, i.e., \( R = 0 \). Then

\[
\frac{Y}{W} = CKG = \frac{Cp}{s+p}
\]

which means the DC gain from \( W \) to \( Y \) is \( C \). Using Final Value Theorem,

\[
y(\infty) = \lim_{s \to 0} Ys = \lim_{s \to 0} \frac{Cp}{s+p} = C \neq 0
\]

Therefore the system is unable to reject constant disturbances.

**Solution 4**

(i) Recall \( T_{r\rightarrow y} = \frac{KP}{1+KP} \). When \( n > 0 \),

\[
0 \neq c = \lim_{s \to 0} \frac{1 - T_{r\rightarrow y}(s)}{s^n} = \lim_{s \to 0} \frac{1+KP}{s^n} = \lim_{s \to 0} \frac{1}{s^n + s^nKP} = \lim_{s \to 0} \frac{1}{s^nK(s)P(s)}
\]

\[
\Leftrightarrow \lim_{s \to 0} s^nK(s)P(s) = \frac{1}{c} \neq 0
\]

When \( n = 0 \),

\[
c = \lim_{s \to 0} (1 - T_{r\rightarrow y}(s)) = \lim_{s \to 0} \frac{1}{1+KP} \implies K(0)P(0) = \frac{1}{c} - 1 < \infty
\]

Also notice that \( K(0)P(0) \neq 0 \), therefore

\[
\lim_{s \to 0} s^nK(s)P(s) = \lim_{s \to 0} nK(s)P(s) = \frac{1}{c} - 1 \neq 0
\]

Hence the system has type \( n \).
(ii) Notice that signal from $W$ to $Y$ can be viewed as with open loop $P$ and feedback $K$. Hence

$$T_{w\to y} = \frac{P}{1 + KP}$$

(iii) Without loss of generality, we can always assume that $T_{w\to y}(s) = s^{k'} A(s) B(s)$ with $k' \in \mathbb{N}_{\geq 0}$ and $A, B$ polynomials with real coefficients such that $A(0) \neq 0$, $B(0) \neq 0$. Notice that

$$\lim_{s \to 0} \frac{T_{w\to y}(s)}{s^k} = \lim_{s \to 0} s^{k'-k} \frac{A(s)}{B(s)} = \frac{A(0)}{B(0)} \lim_{s \to 0} s^{k'-k}$$

If $k' > k$,

$$\lim_{s \to 0} \frac{T_{w\to y}(s)}{s^k} = 0$$

If $k' < k$, $\lim_{s \to 0} T_{w\to y}(s)$ is not defined. Hence we must have $k' = k$. In other words, $T_{w\to y}$ has type $k$ with respect to disturbance inputs if it has a zero of order $k$ at the origin.

(iv) Let $w(t)$ be a degree of $m$ polynomial disturbances. Then $W(s) = \frac{W_0}{s^{m+1}}$. By Final Value Theorem,

$$y(\infty) = \lim_{s \to 0} T_{w\to y}(s) W(s) s = \lim_{s \to 0} s^{k-m} W_0 \frac{A(s)}{B(s)} = \begin{cases} 0 & \text{if } m < k \\ \frac{W_0 A(0)}{B(0)} & \text{if } m = k \\ \text{not defined} & \text{if } m > k \end{cases}$$

Hence the system of type $k$ with respect to disturbances can achieve perfect steady-state rejection of polynomial disturbances of degree $m < k$, but not when $mk$.

(v) Recall $T_{w\to y} = \frac{P}{1 + KP}$.

(a) $$T_P = \frac{P}{1 + KP} = \frac{1}{\frac{s^2 + 1}{1 + KP}} = \frac{1}{s^2 + (KP + 1)}$$

no zero at origin, hence type 0.

(b) $$T_{PD} = \frac{P}{1 + (KP + KD)s} = \frac{1}{\frac{s^2 + 1}{1 + (KP + KD)s}} = \frac{1}{s^2 + KD s + (KP + 1)}$$

no zero at origin, hence type 0.

(c) $$T_{PID} = \frac{P}{1 + (KP + KD s + Ki)} = \frac{1}{\frac{s^2 + 1}{1 + (KP s + KD s^2 + K_i)s}} = \frac{s}{s^3 + K_D s^2 + (KP + 1)s + Ki}$$

a zero at origin, hence type 1.