NOTE: You don’t need to submit this problem set, it is just to help you prepare for the final exam. Solutions will be posted on the web.

Reading: FPE, Sections 7.6 and 7.10.2.

Problems:

1. (exam material) In class we derived the closed-loop system obtained with dynamic output feedback in 
\((x, \hat{x})\)-coordinates:

\[
\begin{pmatrix}
\dot{x} \\
\dot{\hat{x}}
\end{pmatrix} =
\begin{pmatrix}
A & -BK \\
LC & A - LC - BK
\end{pmatrix}
\begin{pmatrix}
x \\
\hat{x}
\end{pmatrix}
\]

and later rewrote it in \((x, e)\)-coordinates. Rewrite the same system in \((\hat{x}, e)\)-coordinates.

2. (exam material) Consider the plant transfer function \(G(s) = \frac{1}{s(s + 1)}\).

a) Find any controllable and observable state-space realization of \(G(s)\).

b) Stabilize the state-space system from part a) by dynamic output feedback. Select arbitrary controller
and observer poles such that the closed-loop system is stable and has reasonable damping (in your judg-
ment).

c) Compute the transfer function of the controller you found in part b). Write it in the form \(kD(s)\), where
\(k\) is a scalar gain (not to be confused with the state feedback gain matrix \(K\)) and \(D(s)\) is a ratio of monic
polynomials (leading coefficients equal 1).

d) Draw the (positive) root locus for \(L(s) = D(s)G(s)\) and find on it the locations of the closed-loop poles
you chose in part b).

e) Draw the Bode plot for \(kD(s)G(s)\) and compute the gain margin and phase margin.

f) Decide whether you’re happy with the closed-loop system. If not, go back and improve the design.

3. (not exam material) Consider the system

\[
\begin{align*}
\dot{x}_1 &= x_1 + x_2 \\
\dot{x}_2 &= -x_1 + x_2 + u \\
y &= 2x_1 + x_2
\end{align*}
\]

and suppose that the control objective is to minimize the performance index \(\int_0^\infty [\rho y^2(t) + u^2(t)] dt, \rho > 0\).

a) Show graphically the locations of the optimal closed-loop poles as the parameter \(\rho\) varies (symmetric
root locus).

b) See why in the limit as \(\rho \to 0\) (“expensive control” case), the optimal closed-loop poles become mirror
images of the open-loop poles across the imaginary axis.

c) See why in the limit as \(\rho \to \infty\) (“cheap control” case), one optimal closed-loop pole cancels the open-loop
zero and the other moves off to \(-\infty\).