Reading: FPE, Sections 6.7.3–6.7.6, 6.3.1 (all editions).

Problems:

This homework asks you to check and improve two control designs given in class on Tuesday, Oct 25. MATLAB use is allowed but you must explain all steps and justify all answers.

1. For the system

$$G(s) = \frac{10}{(\frac{s}{0.2}+1)(\frac{s}{0.5}+1)}$$

we want to design a lag controller that provides PM of at least 60° and steady-state tracking of constant references within 10%.

a) For the controller derived in class:

$$KD(s) = 0.4\frac{s+0.05}{s+0.02}$$

compute the PM and steady-state tracking error to verify that the specs are met.

b) Suppose that to get better damping, we increase the PM spec to 70°, while keeping the same steady-state tracking spec as in a). Modify the design to achieve the new specs. Verify that your design indeed works.

Solution:

$$G(s) = \frac{10}{\left(\frac{s}{0.2}+1\right)\left(\frac{s}{0.5}+1\right)}$$

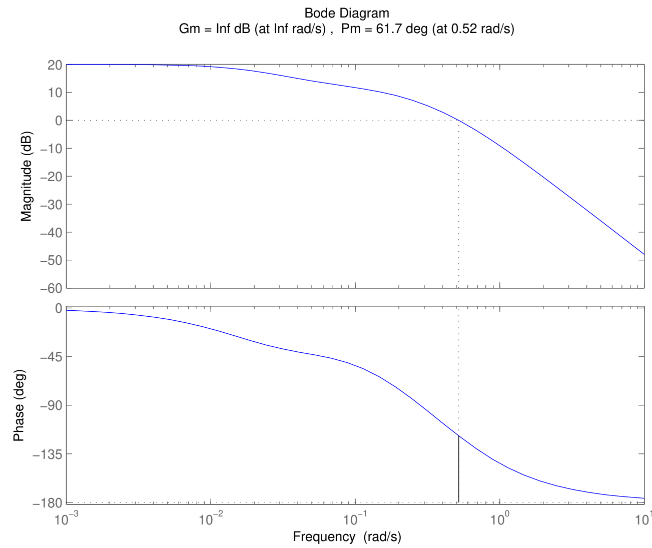
Bode plot is attached (for reference).

(a)

$$KD(s) = 0.4\frac{s+0.05}{s+0.02}$$

In the attached figure, we can see that PM = 61.7 deg, $\omega_c = 0.52$, and GM $= \infty$. For steady-state error for constant input:

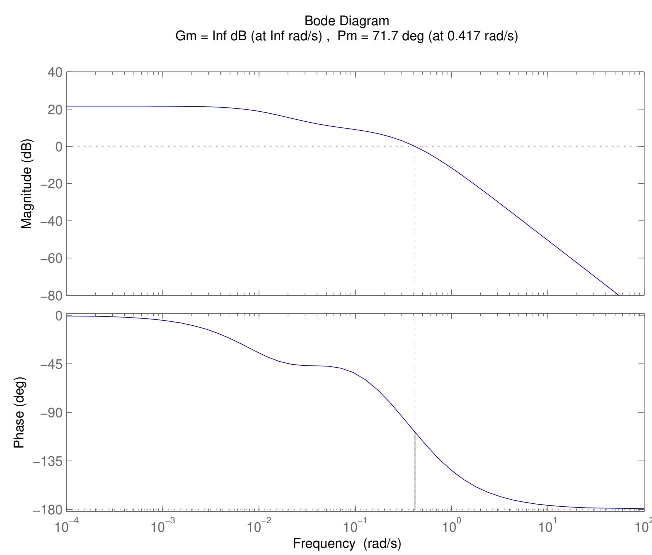
$$e(\infty) = \left.\frac{1}{1+KD(s)G(s)}\right|_{s=0} = \frac{1}{1+0.4\times 2.5\times 10} = \frac{1}{11} < 0.1.$$

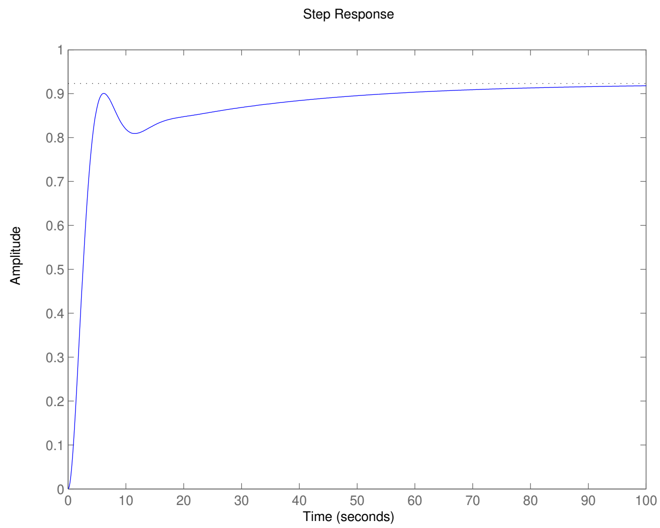


(b) In the first glance, it appears that we need to bring ω_c to ≈ 0.4 for PM $= 70^\circ$ and that can be achieved by changing the controller gain from 0.4 to 0.3. But this will increase the error from $\frac{1}{11}(< 0.1)$ to $\frac{1}{8.5}$, which contradicts the design spec. Therefore, we change the $\frac{z}{p}$ ratio for lag from 2.5 to 4 and bringing down the zero to 0.4 (1 decade less than new ω_c) and out new controller will be

$$KD(s) = 0.3 \cdot \frac{s + 0.04}{s + 0.01} \Rightarrow e(\infty) = \frac{1}{13} < 0.1.$$

Bode plot for PM, GM as well as time response plot are attached.





2. For the same system as in problem 1, we want to design a lead/lag controller that provides bandwidth of at least 2, PM of at least 60°, and steady-state tracking of constant references within 1%.

a) For the controller derived in class:

$$KD(s) = 4\frac{\frac{s}{0.8}+1}{\frac{s}{5}+1} \cdot \frac{s+0.05}{s+0.02}$$

compute the PM, bandwidth, and steady-state tracking error to verify if the specs are met.

b) Suppose that in addition to the above specs, the bandwidth cannot exceed 6. Modify the design to incorporate this new spec, and verify that it indeed works.

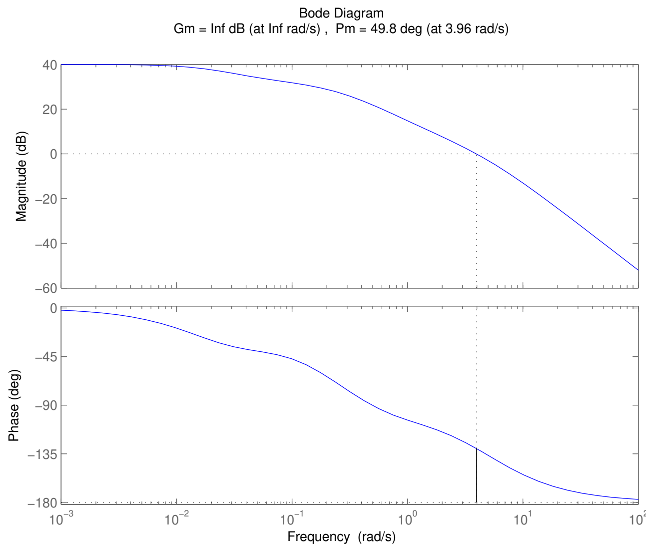
Solution:

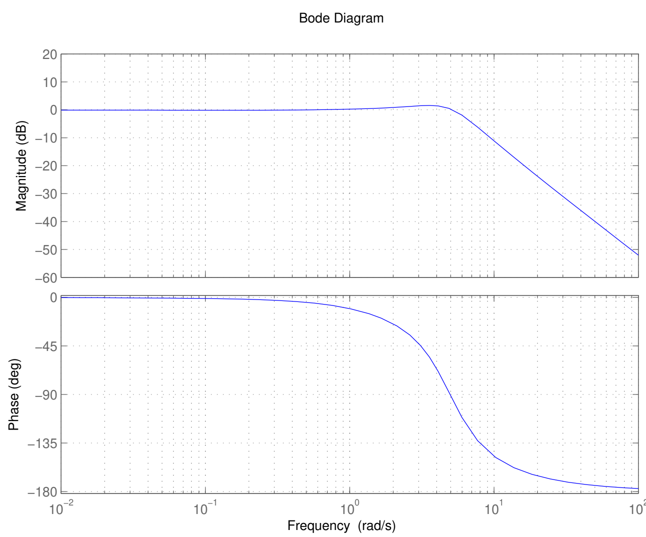
$$KD(s) = 4 \cdot \frac{\frac{s}{0.8}+1}{\frac{s}{5}+1} \cdot \frac{s+0.05}{s+0.02}.$$

(a) The attached Bode shows that PM $\approx 50°$ with $\omega_c = 4$ so the phase-margin is NOT achieved. The error is

$$e(\infty) = \frac{1}{1+KD(0)G(0)} = \frac{1}{101} < 0.01$$

and the closed-loop bode shows that BW ≈ 6.36.



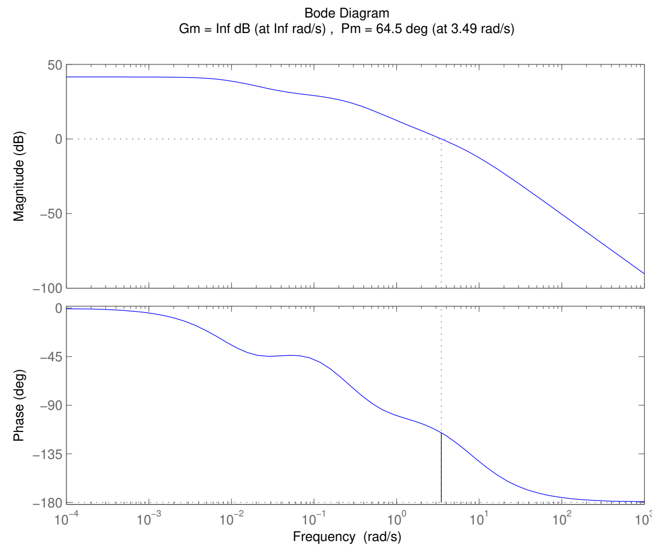


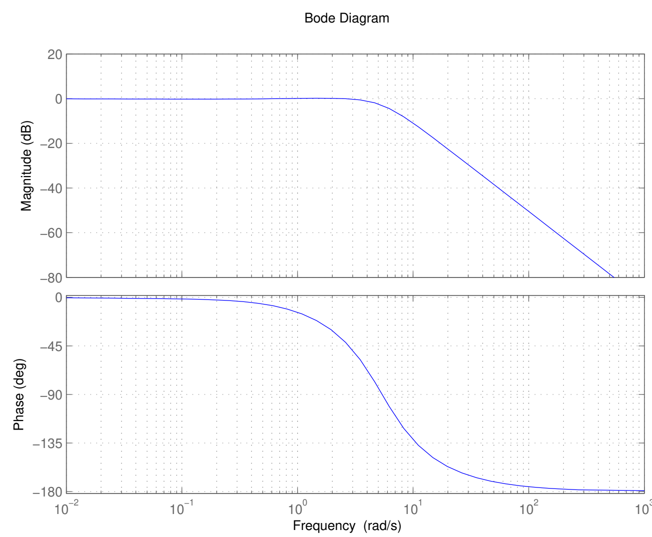
(b) To bring down BW (it's > 6), we need to reduce ω_c, also we need to increase our PM so we start with the lag controller from part 1(b) and pull the lead pole a little further to increase the phase which is provided by the lead controller and adjust the gain for error requirements:

$$KD(s) = 3 \cdot \frac{\frac{s}{0.8} + 1}{\frac{s}{8} + 1} \cdot \frac{s + 0.04}{s + 0.01}.$$

The attached Bodes show that PM $= 62^\circ$ and BW $= 5.34$ and

$$e(\infty) = \left.\frac{1}{1 + KD(s)G(s)}\right|_{s=0} = \frac{1}{121} < 0.01.$$


Bode Diagram
Gm = Inf dB (at Inf rad/s) , Pm = 64.5 deg (at 3.49 rad/s)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)