ECE 486, Fall 2023 **PROBLEM SET 5** **Due Thursday, *Oct 5***
(one week after the midterm)

Reading: FPE, Sections 5.1, 5.2, 5.6.1.

Problems:

1. Consider the plant with transfer function $L(s) = \dfrac{1}{s^2 + 2s}$. Under the action of a constant feedback gain K, the closed-loop poles are the roots of the characteristic polynomial $s^2 + 2s + K$.

a) Draw the (positive) root locus. (Use the expression for the closed-loop poles in terms of K obtained via the quadratic formula.)

b) Consider the settling time spec $t_s \leq 4$. Give some value (or range of values) of K for which the closed-loop system meets this spec. Justify your choice. Show the corresponding pole locations on the root locus.

c) Consider the rise time spec $t_r \leq 1$. Give some value (or range of values) of K for which the closed-loop system meets this spec. Justify your choice. Show the corresponding pole locations on the root locus.

d) Consider the overshoot spec $M_p \leq 0.1$. Give some value (or range of values) of K for which the closed-loop system meets this spec. Justify your choice. Show the corresponding pole locations on the root locus.

e) Suppose that it is desired to place the closed-loop poles at $-1 \pm j$. Find the value of K that will achieve this, using the characteristic equation $s^2 + 2s + K = 0$ *but without using the quadratic formula.* (In other words, you should find a way of doing this that would also work for a higher-order example.)

2. Consider the following transfer functions:

$$1)\ L(s) = \frac{1}{s(s^2 + 4s + 8)} \qquad\qquad 2)\ L(s) = \frac{s}{(s-1)(s+1)^2}$$

For each one of these, do the following:

a) Mark the zeros and poles on the s-plane and use Rule 2 from class to plot the real-axis part of the root locus.

b) Use the phase condition from class to test whether or not the point $s = j$ is on the root locus. If you run into "non-obvious" angles, *estimate* rather than *calculate* them, this should be enough.

c) Apply Rules 3 and 4 to determine asymptotes and departure and arrival angles. Plot the root locus branches based on this information.

d) Apply Rule 5 to determine imaginary-axis crossings (if any), and complete the (positive) root locus by using Rule 6 to check for multiple roots.

e) Plot the (positive) root locus using the MATLAB `rlocus` command.

f) Repeat items a)–e) for the *negative* root locus.

Turn in your MATLAB plots as well as hand sketches of root loci along with all accompanying calculations and explanations.