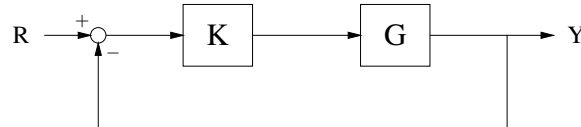**Reading:** FPE, Sections 4.1–4.3 (the material not discussed in class is optional).

Problems:

1. Consider the following feedback system, where K is a constant gain and $G(s) = \dfrac{1}{s^3 + 2s^2 + 2s + 1}$:

Using the Routh-Hurwitz criterion, show that for $-1 < K < 3$ the system is stable but for $K \geq 3$ the system is unstable. (This illustrates the destabilizing effect of feedback when the gain is too high.)

2. The goal of this exercise is to compare sensitivity of open-loop and closed-loop control with respect to errors in the *control gain*. Consider the DC motor model discussed in class, with no disturbance ($w_\ell = 0$). Let the control gain sensitivity be defined as follows: when the controller gain changes from K to $K + \delta K$ and, as a result, the steady state gain (DC gain) of the overall system changes from T to $T + \delta T$, we define $S_K = \dfrac{\delta T/T}{\delta K/K}$. (The motor gain A remains fixed here.)

a) Compute the sensitivity S_K in the open-loop case, starting from the nominal values $K_{ol} = 1/A$ and $T_{ol} = 1$.

b) Compute the sensitivity S_K for a feedback gain K_{cl}, using the approximate formula $\delta T = \dfrac{dT}{dK}\delta K$ and the fact that the nominal system gain is, as derived in class, $T_{cl} = \dfrac{AK_{cl}}{1 + AK_{cl}}$.

Hint: your final answers in a) and b) should be the same as the ones derived in class for sensitivity to errors in the motor gain A.

3. Suppose that the DC motor discussed in class is connected in feedback with a PI controller $k_P + k_I/s$. (This refers to the standard negative unity feedback control configuration, where the input to the controller is $e = r - y = \omega_{ref} - \omega_m$.) Write down the full transfer function of the closed-loop system in the presence of load/disturbance w_ℓ. (For $k_I = 0$ this should match what we derived in class for constant feedback gain.) Is it true that by proper choice of gains k_P and k_I we can achieve arbitrary pole placement as well as perfect constant reference tracking and constant disturbance rejection in steady state? Justify your answer.

4. Consider again the standard feedback configuration like the one in Problem 1, but with $K(s)$ and $G(s)$ unknown transfer functions. Suppose that the transfer function from R to Y is $\dfrac{Y(s)}{R(s)} = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ for some $\zeta, \omega_n > 0$.

a) Based on this information, find the forward gain $K(s)G(s)$.

b) Determine the system type and discuss what it implies about the system's steady-state tracking ability.