ECE 486, Fall 2023 **PROBLEM SET 3** **Due Thursday, Sep 14**

Reading: FPE, Sections 3.3–3.6.

Problems: (unless otherwise noted, you can use a calculator/computer to arrive at numerical answers)

1. Using techniques for block diagram reduction discussed in class, find the transfer functions of the systems a) and d) shown on the next page (taken from the textbook). Of the four diagrams shown, only do the first and the last.

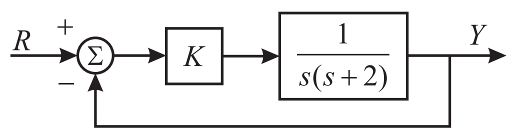
2. Consider the transfer function $H(s) = \dfrac{1}{s^2+s+1}$.

a) Suppose that you are given the following time-domain specs: $t_r \le 2.5$, $t_s \le 8$. Plot the admissible pole locations in the s-plane corresponding to these two specs. Does the given system satisfy these specs?

b) Suppose that *in addition to* the specs from a), we have the following spec on the overshoot: $M_p \le 1/e^2$. Plot the admissible pole locations in the s-plane corresponding to all three specs. Does the given system satisfy the new spec?

c) Now suppose that you are given the two specs from a) *plus* the following spec on the peak time: $t_p \le 4$ (*instead of* the overshoot spec). Plot the admissible pole locations in the s-plane corresponding to these three specs. (Pole locations for peak time were not discussed in class, so you need to derive this.) Does the given system satisfy the new spec?

3. For the feedback system shown in the diagram



determine the range of proportional gains K so that the overshoot of the closed-loop system (in response to the unit step reference input) is no more than 10%.

4. The purpose of this exercise is to illustrate that a system with a pair of poles on the imaginary axis is not stable because it gives unbounded response to a sinusoidal signal that matches the system's natural frequency (this phenomenon is known as *resonance*). Consider the transfer function

$$G(s) = \frac{1}{s^2+\omega_n^2}, \qquad \omega_n > 0$$

and the input $u(t) = \sin(\omega t)$, $\omega > 0$.

a) Derive the response of the system, using both the partial fractions method and the frequency response formula. What happens to this response as ω approaches ω_n? (We know that the two methods give different answers, the first one being more precise in terms of the transient response, but qualitatively they should lead to the same conclusion here.)

b) To characterize the response more precisely, compute it using the convolution integral $y(t) = \int_0^t h(t-\tau)u(\tau)d\tau$ (note: these are the correct limits of integration). Identify the term in your calculation that leads to unbounded response when $\omega = \omega_n$. You can use the trigonometric identities $\sin(a-b) = \sin a \cos b - \cos a \sin b$ and $(\sin a)^2 = (1-\cos 2a)/2$.

5. Determine whether or not the following polynomials have any RHP roots:

a) $s^4 + 8s^3 + 32s^2 + 80s + 100$ b) $s^5 + 5s^4 + 2s^3 - s^2 + 4s + 10$

(Computer use not allowed.)

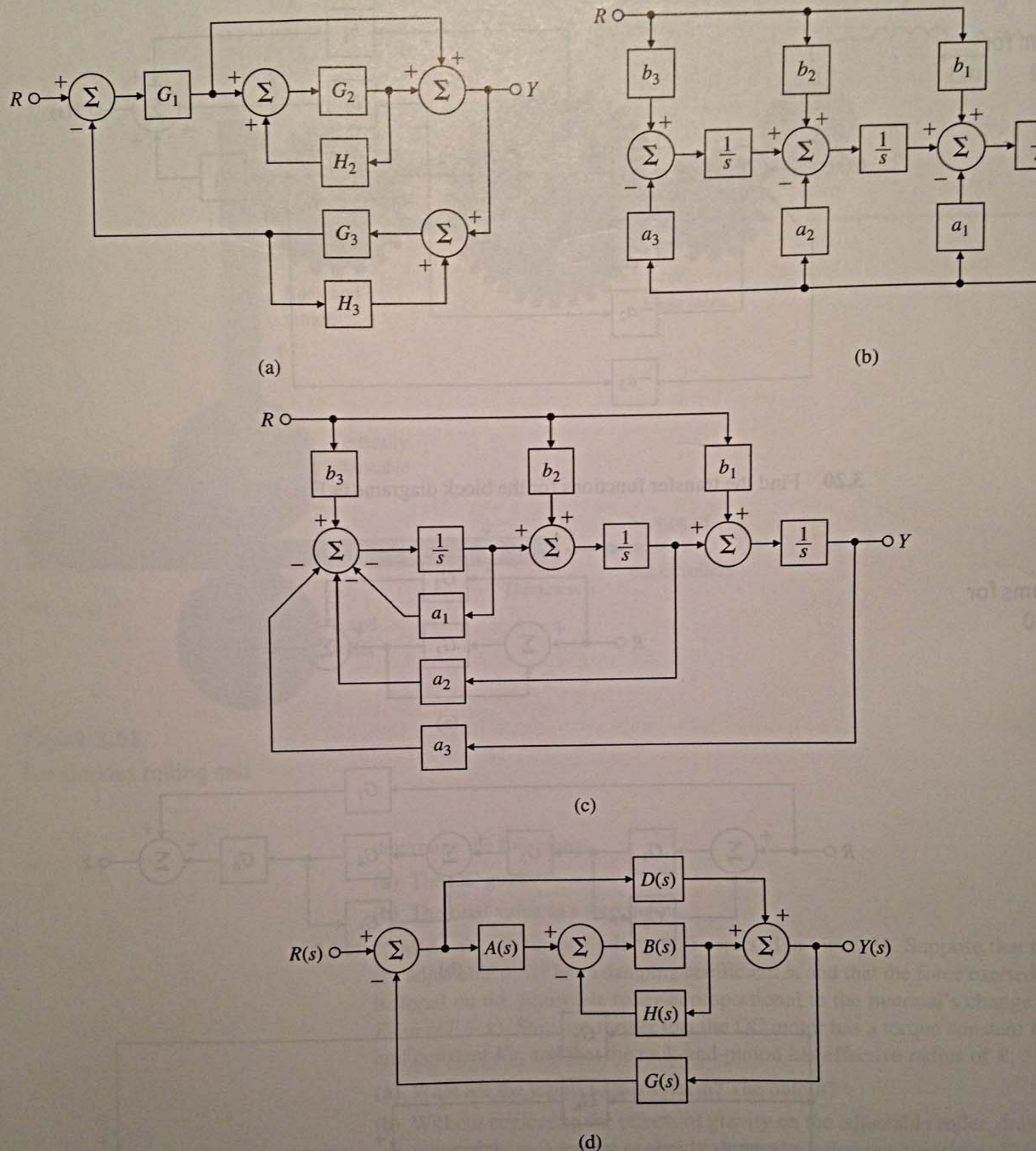


Figure 3.54
Block diagrams for Problem 3.21