ECE 486, Fall 2023 **PROBLEM SET 1** **Due Thursday, Aug 31**

Reading: FPE (Franklin, Powell, Emami-Naeini, 6th or 7th edition), Sections 1.1, 1.2, 2.1–2.4, 7.2, 9.2.1.

Problems: (the first two problems are designed to test your background)

1. For each matrix and/or vector pair given below, compute their product $A \cdot B$ if possible, or explain why it is not possible.

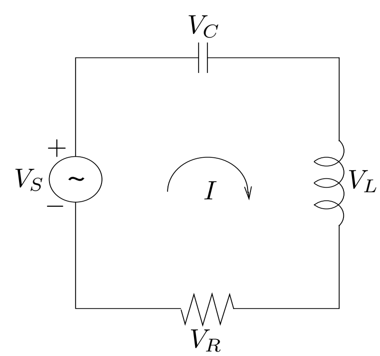
a) $A = \begin{pmatrix} 0 & 3 \\ -1 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ -5 & 1 \end{pmatrix}$; b) $A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 2 & -3 \end{pmatrix}, B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$; c) $A = \begin{pmatrix} 2 & 3 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 2 & -2 \\ 3 & 0 \end{pmatrix}$.

2. Compute the magnitude and the phase of the following complex numbers:

a) $1 - 2j$ b) $2 + 3j$ c) $(1 - 2j) \cdot (2 + 3j)$

How do the answers for c) relate to those for a) and b)? State the general rule behind this.

3. Derive a state-variable model, of the form $\dot{x} = Ax + Bu$, for the following circuit:



(Use the capacitor law $I = C\dot{V}_C$.) Note that you have to decide which variables to take as the states and which one to take as the input. Make sure to declare your choice.

4. Convert each of the following high-order differential equations into the state-variable form:

a) $\ddot{x} + 2\dot{x} - 0.5x = 2u$ b) $x^{(4)} + \ddot{x} + x = u$ ($x^{(4)}$ is the 4th derivative of x with respect to time)

5. Derive the linearization of the equation $\dot{x} = \cos x$ around the equilibrium point $x = \pi/2$ using the following two methods:

a) Use the linear Taylor approximation $f(x) \approx f(x_0) + f'(x_0) \cdot (x - x_0)$ with $x_0 = \pi/2$ to obtain the linearized equation in the form $(x \dot{-} x_0) = f'(x_0) \cdot (x - x_0)$. Note: $\dot{x}$ and $(x \dot{-} x_0)$ are the same (since x_0 is constant) but we want to express the linearized equation in terms of the *deviation from the equilibrium.*

b) Make the coordinate shift $z = x - \pi/2$, linearize the system $\dot{z} = \cos(z + \pi/2)$ around $z = 0$, and then re-express the resulting equation in terms of x.

Make sure the two results agree. Provide a graphical explanation of why the linear function that you found approximates $\cos x$ well near $x = \pi/2$.