

ECE 486: Control Systems

Lecture 7C: Proportional (P) Control

Key Takeaways

This lecture describes closed-loop proportional control. The controller sets the plant input proportional to the error.

Closed-loop control can achieve higher performance but requires a sensor.

Larger proportional gains:

- (i) improve reference tracking and disturbance rejection
- (ii) increase the closed-loop speed of response

but:

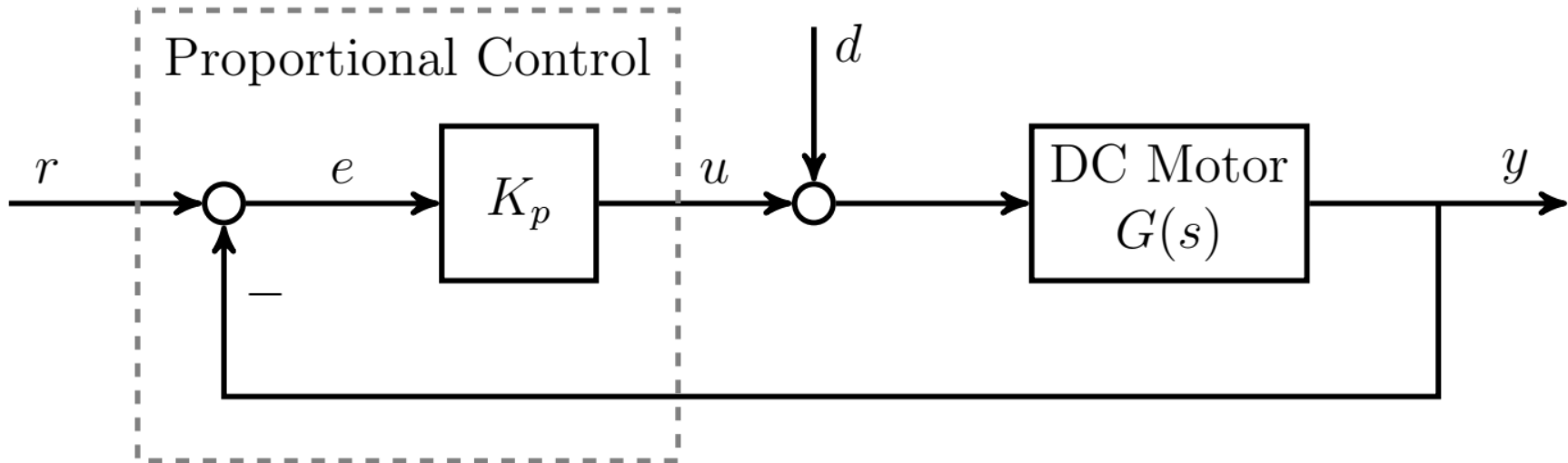
- (i) require larger control inputs
- (ii) can excite unmodeled dynamics

Proportional (P) Control

Closed-loop, proportional control for DC motor:

1. User specifies the desired motor speed, $r(t)$
2. Controller computes the tracking error $e(t) = r(t) - y(t)$
3. Controller sets input voltage to $u(t) = K_p e(t)$ where K_p is a gain to be selected.

Closed-loop control requires a sensor to measure y .



Model for Closed-Loop Control

Recall the first-order model for the motor:

$$\dot{y}(t) + a_0 y(t) = b_0 u(t) + b_0 d(t)$$

where: $a_0 = 0.94 \frac{1}{\text{sec}}$ and $b_0 = 766.8 \frac{\text{rad}}{\text{sec}^2 \text{V}}$

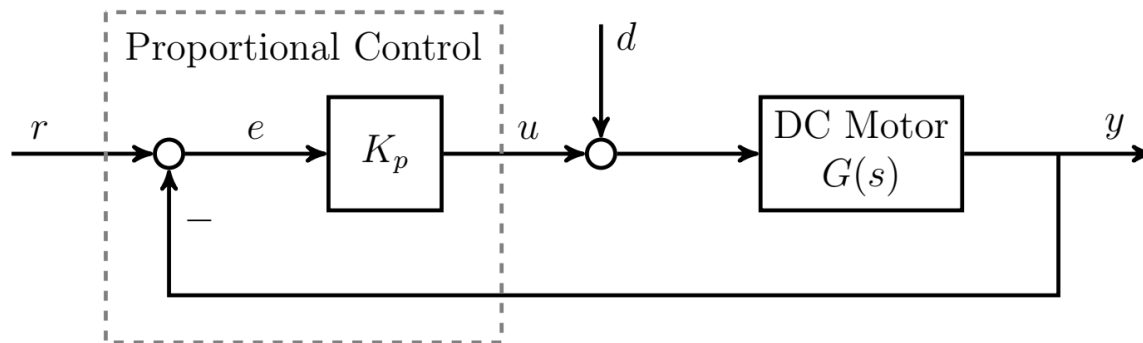
$$G(s) = \frac{b_0}{s + a_0}$$

Substitute $u = K_p (r - y)$ into the model:

$$\dot{y}(t) + a_0 y(t) = b_0 K_p (r(t) - y(t)) + b_0 d(t)$$

Bring term involving y to the left side:

$$\dot{y}(t) + (a_0 + b_0 K_p) y(t) = (b_0 K_p) r(t) + b_0 d(t)$$



Closed-Loop Response

The dynamics of the closed-loop system are:

$$\dot{y}(t) + (a_0 + b_0 K_p) y(t) = (b_0 K_p) r(t) + b_0 d(t)$$

The response has the following properties:

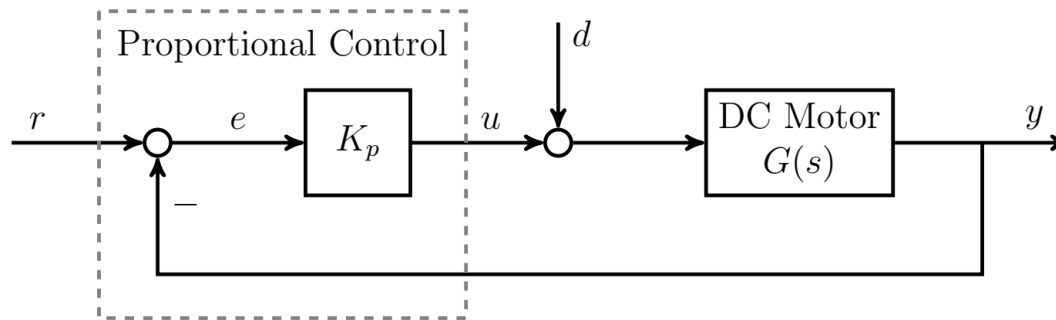
1. The system has a pole at $s = -(a_0 + b_0 K_p)$ with $\tau = \frac{1}{a_0 + b_0 K_p}$.

Larger values of K_p yield faster closed-loop dynamics.

2. If $r(t) = \bar{r}$ and $d(t) = \bar{d}$ then:

$$y(t) \rightarrow \bar{y} := \frac{b_0 K_p}{a_0 + b_0 K_p} \bar{r} + \frac{b_0}{a_0 + b_0 K_p} \bar{d} \text{ as } t \rightarrow \infty$$

We get $\bar{y} \approx \bar{r}$ if K_p is "large" even if there are disturbances, but there will always be some error in steady-state.

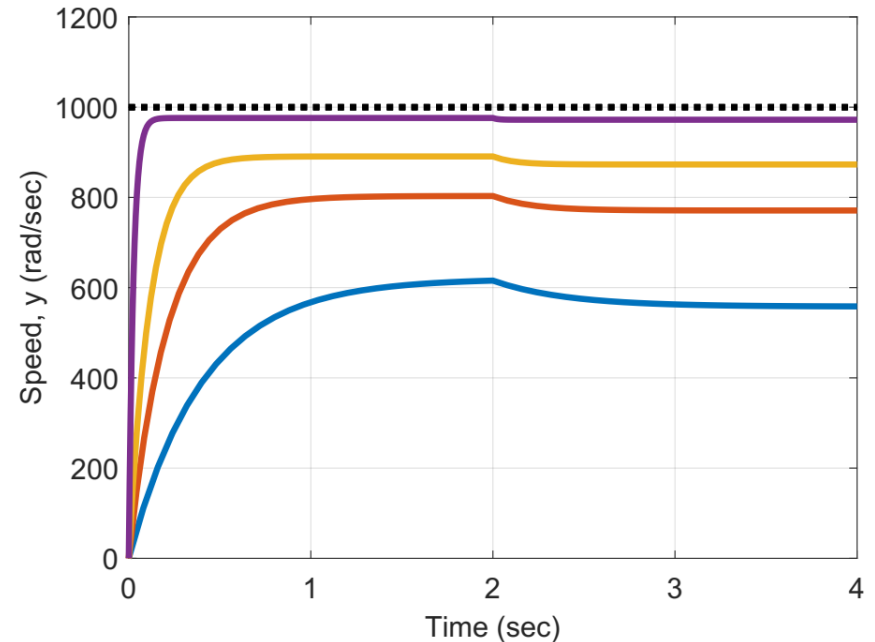
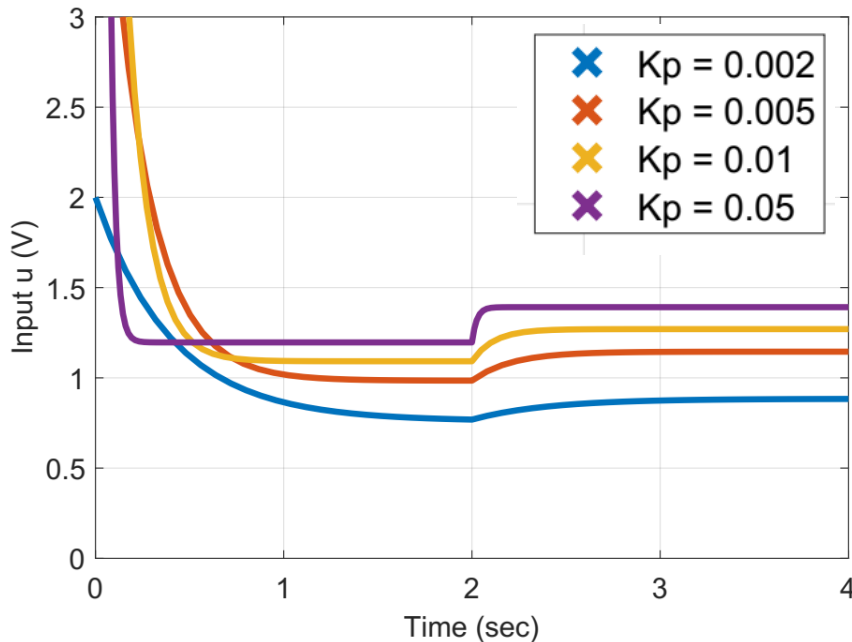


Response of Closed-Loop System

Simulations with different gains K_p and:

- $r(t) = 1000 \frac{rad}{sec}$
- $d(t) = -0.2V$ for $t \geq 2$ sec

Larger gains yield: (i) faster response, (ii) better tracking, but larger control inputs.

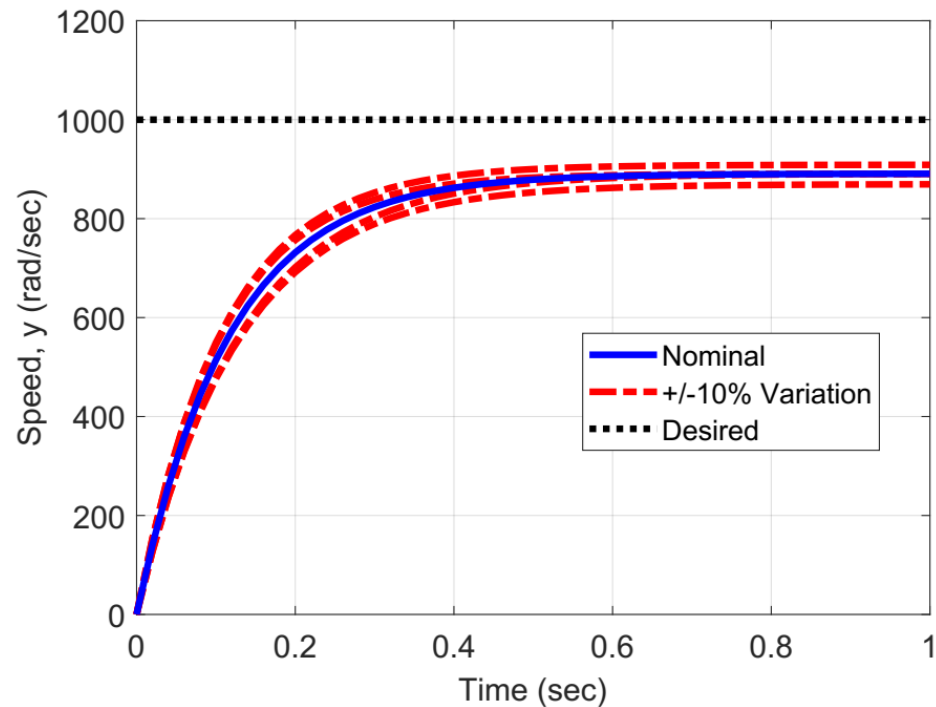
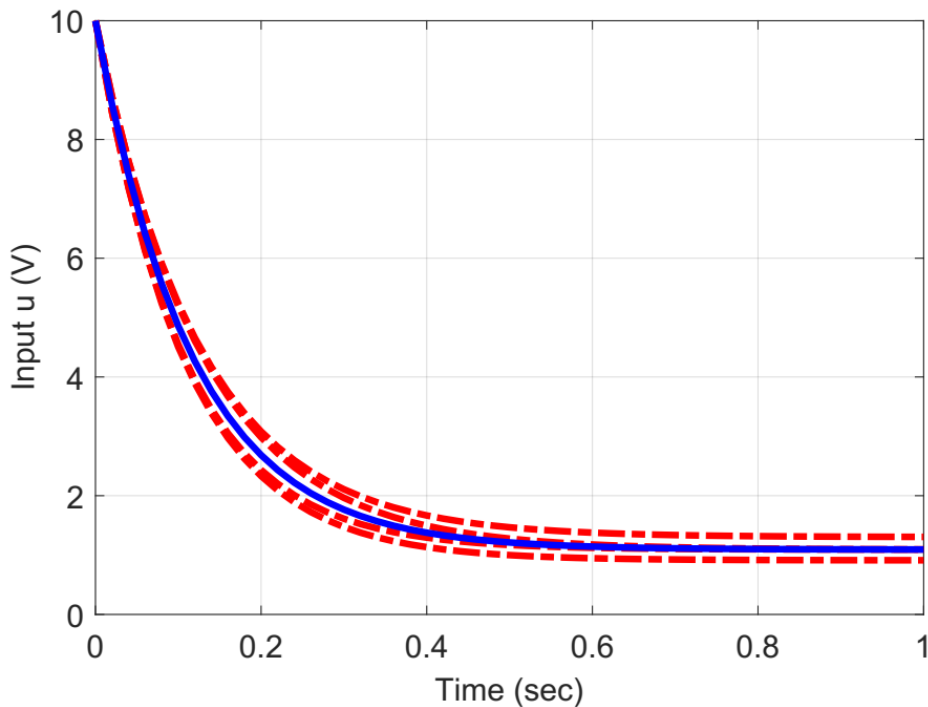


Impact of Model Variations

Simulations with $K_p=0.01$ and:

- $r(t) = 1000 \frac{\text{rad}}{\text{sec}}$
- $\pm 10\%$ variation in (a_0, b_0) in plant dynamics.

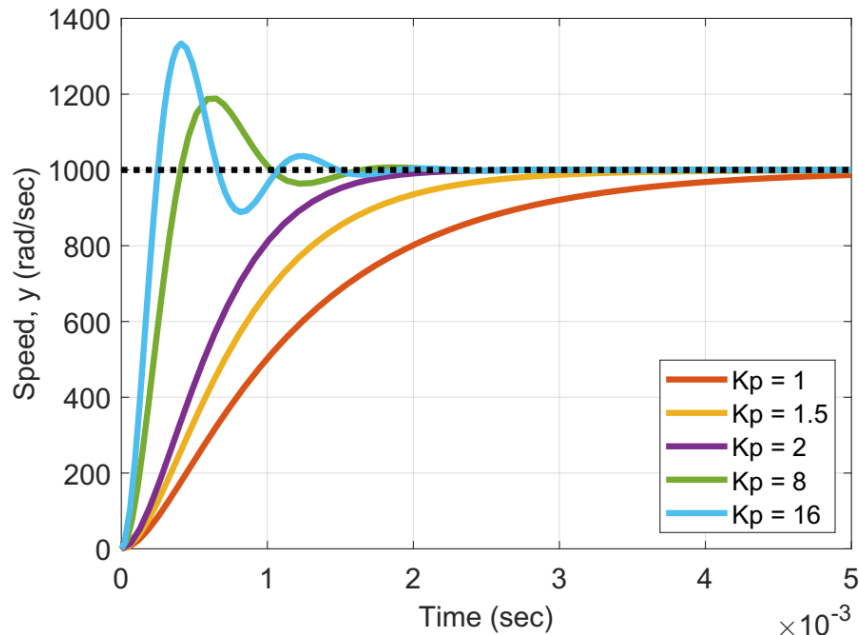
Proportional control is less sensitive to model variations.



Impact of Unmodeled Dynamics

Simulations with different gains K_p and:

- $r(t) = 1000 \frac{\text{rad}}{\text{sec}}$
- Second-order model including “fast” electrical dynamics



Impact of Unmodeled Dynamics

Simulations with different gains K_p and:

- $r(t) = 1000 \frac{\text{rad}}{\text{sec}}$
- Second-order model including “fast” electrical dynamics

Unmodeled dynamics will degrade performance if excited by the controller.

