ECE 486: Control Systems

Lecture 6A: Effect of Extra Poles & Zeros

Key Takeaways

This lecture considers the effect of extra poles and zeros on the step response.

LHP Poles: Increase settling time.

The effects are small if the pole is far in the LHP.

LHP Zeros: Increase overshoot, decrease rise time, and have no effect on settling time.

The effects are small if the zero is far in the LHP.

RHP Zeros: Cause undershoot but no effect on settling time.

The effects are small if the zero is far in the RHP.

First-Order Step Response

Step Response:

$$\dot{y}(t) + 1.5y(t) = 1.5u(t)$$

with $y(0) = 0$ and $u(t) = 1$ for $t \geq 0$

$$G(s) = \frac{1.5}{s+1.5}$$

1. Stable: $s + 1.5 = 0 \quad \Rightarrow \quad s_1 = -1.5 < 0$

2. Time constant: $\tau = \frac{1}{|s_1|} = \frac{1}{1.5} = \frac{2}{3} sec$

3. Settling time: $3\tau = 2sec$

4. Final Value: $\bar{y} = G(0)\bar{u} = 1$

First-Order Step Response

Step Response:

$$\dot{y}(t) + 1.5y(t) = 1.5u(t)$$
$$\text{with } y(0) = 0 \text{ and } u(t) = 1 \text{ for all } t \geq 0$$

$$G(s) = \frac{1.5}{s+1.5}$$

Response:

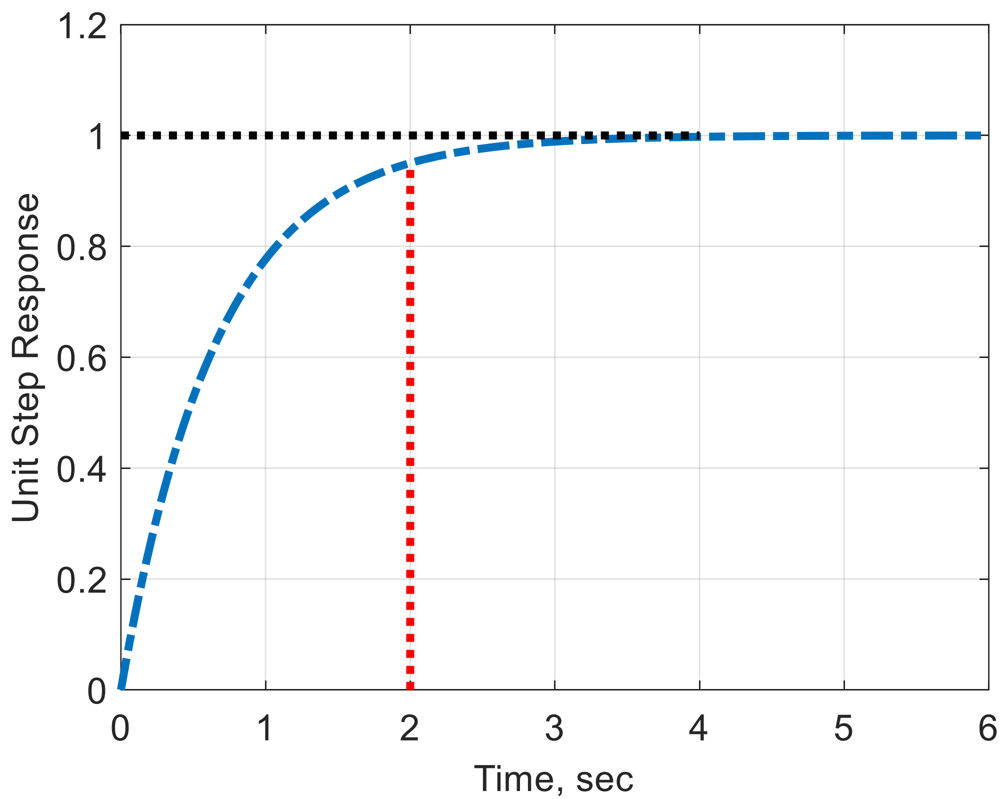
(i) stable,

(ii) 3τ = 2sec,

(iii) $\bar{y}$= 1

Matlab:

```
>> G=tf(1.5,[1 1.5]);
>> [yunit,t]=step(G);
>> plot(t,yunit);
```

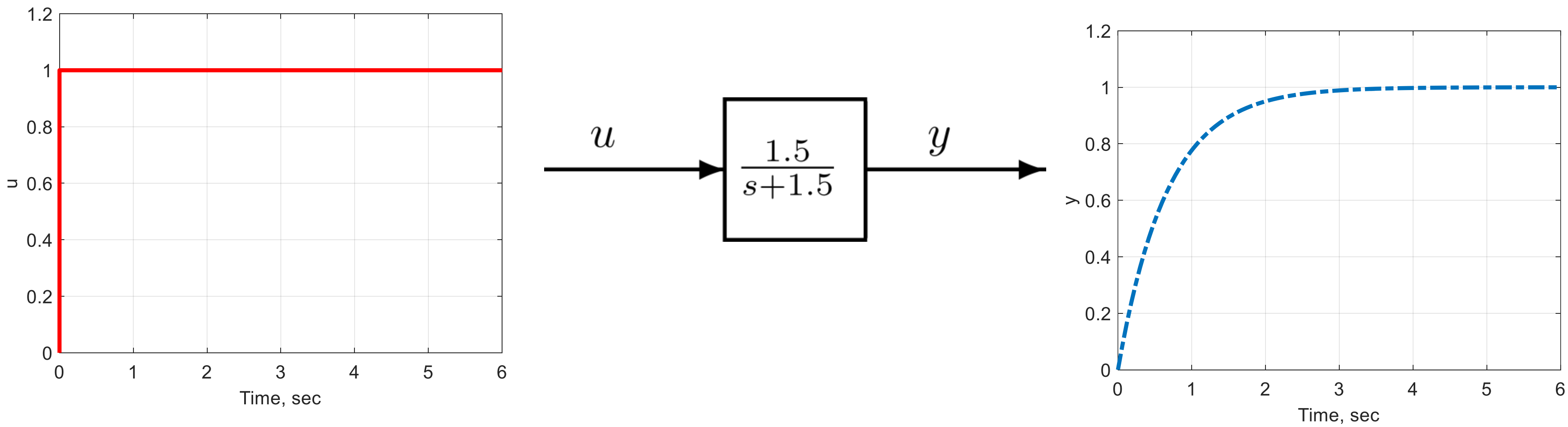


Additional Pole

Consider the second-order system:

$$\ddot{y}(t) + (1.5 + p)\,\dot{y}(t) + (1.5p)\,y(t) = (1.5p)\,u(t)$$

$$G_p(s) = \frac{1.5p}{s^2+(1.5+p)s+1.5p} = \frac{p}{s+p} \cdot G(s) \text{ where } G(s) = \frac{1.5}{s+1.5}$$



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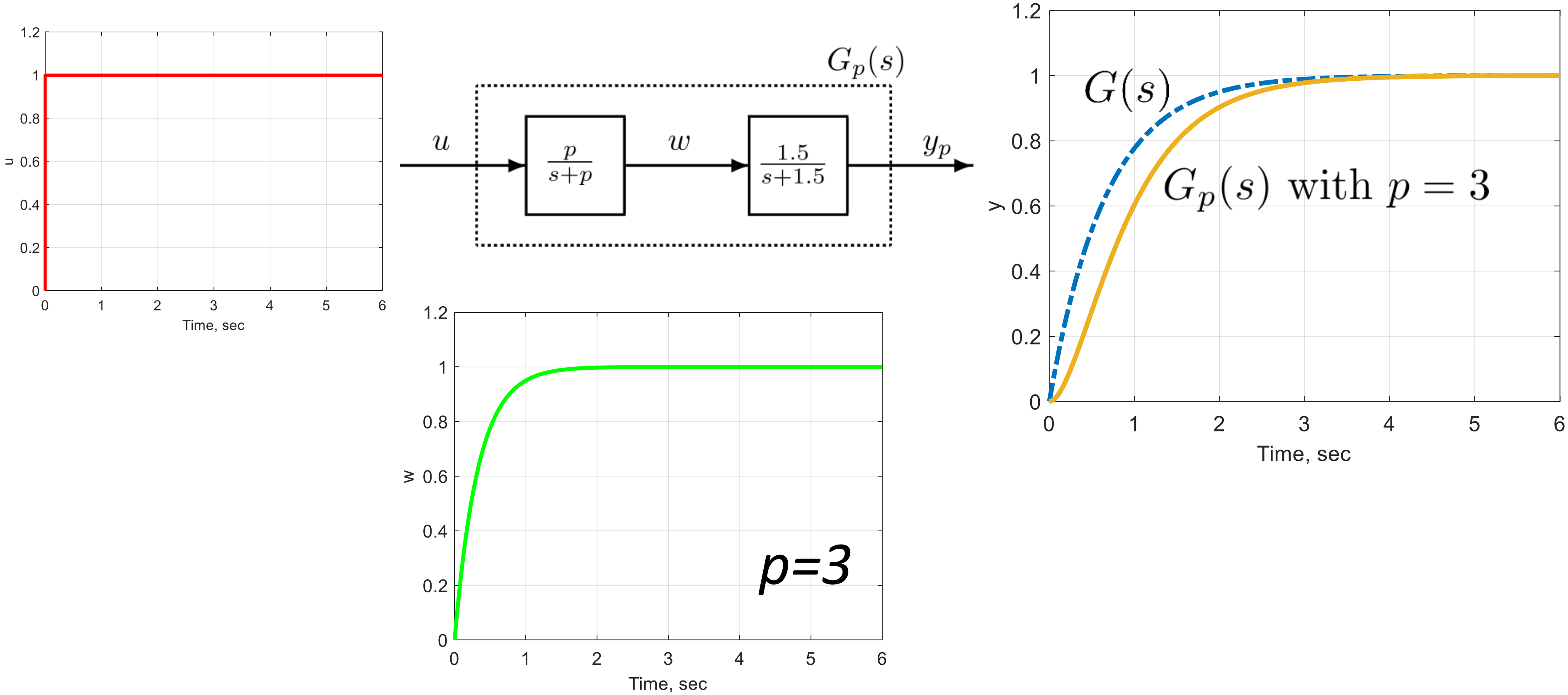

Additional Pole

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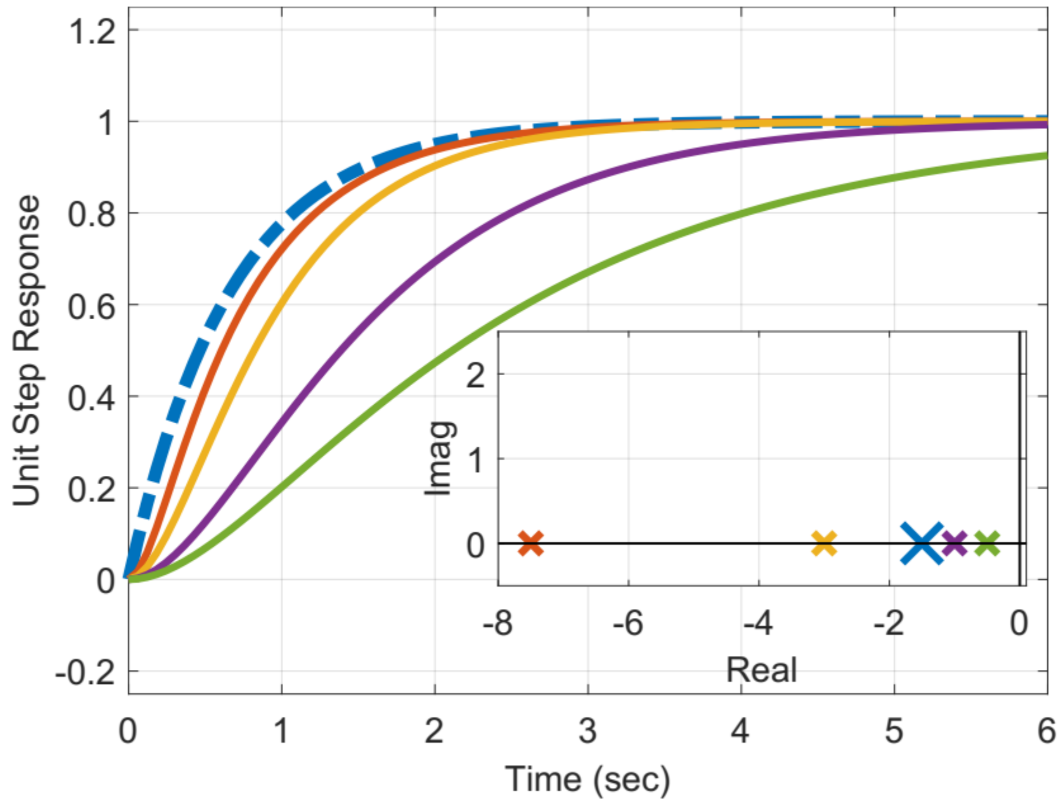
Additional poles in the LHP increase the settling time.

The effect is small if the extra pole is far in the LHP (≈ 5 ×faster than slowest pole)



Second-Order Step Response

Step Response:

$\ddot{y}(t) + 2\dot{y}(t) + 4y(t) = 4u(t)$

with $y(0) = 0, \dot{y}(0) = 0$ and $u(t) = 1$ for $t \geq 0$

$$G(s) = \frac{4}{s^2+2s+4}$$

1. Underdamped and Stable:

$$\omega_n = 2\frac{rad}{sec} \text{ and } \zeta = 0.5 \quad \Rightarrow \quad s_{1,2} = -1 \pm 1.73j$$

2. Settling time: $T_s = 3\tau = \frac{3}{|Re(s_{1,2})|} = 3sec$

3. Final Value: $\bar{y} = G(0)\bar{u} = 1$

4. Peak Overshoot:

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \approx 0.16$$
$$\Rightarrow y(T_p) = (1 + M_p)\bar{y} \approx 1.16$$

Second-Order Step Response

Step Response:

$$\ddot{y}(t) + 2\dot{y}(t) + 4y(t) = 4u(t)$$

with $y(0) = 0, \dot{y}(0) = 0$ and $u(t) = 1$ for $t \geq 0$

$$G(s) = \frac{4}{s^2+2s+4}$$

Response:

(i) stable, underdamped

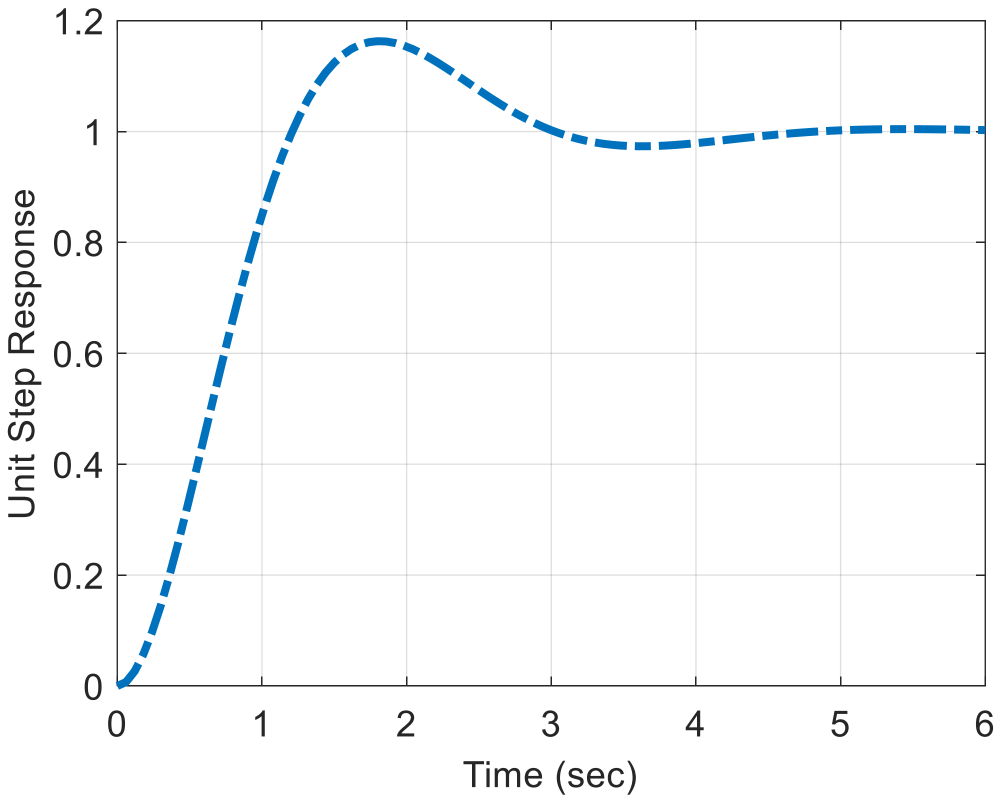
(ii) 3τ = 3sec,

(iii) $\bar{y}$= 1

(iv) $y(T_p) \approx 1.16$

Matlab:

```
>> G=tf(4,[1 2 4]);
>> [yunit,t]=step(G);
>> plot(t,yunit);
```

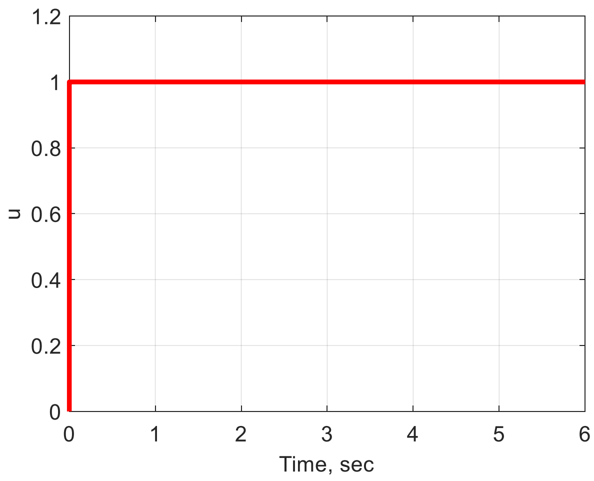


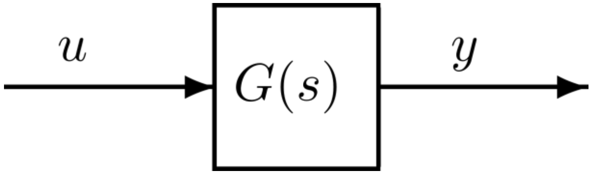
Effect of a Zero

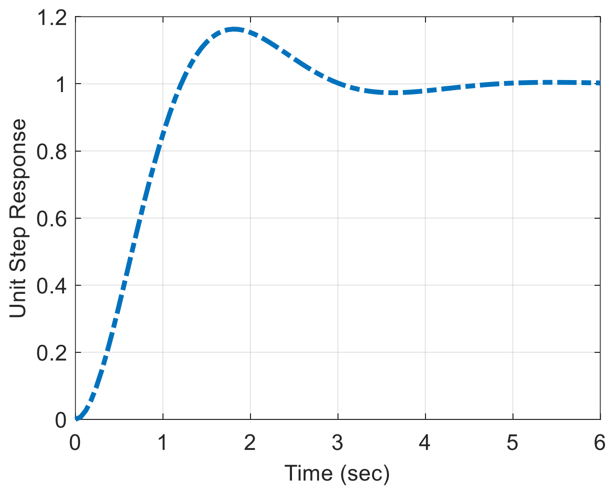
Consider the second-order system:

$$\ddot{y}(t) + 2\,\dot{y}(t) + 4\,y(t) = (4b)\,\dot{u}(t) + 4u(t)$$

$$G_b(s) = \frac{4b\,s+4}{s^2+2s+4} = (bs+1)\cdot G(s) \text{ where } G(s) = \frac{4}{s^2+2s+4}$$







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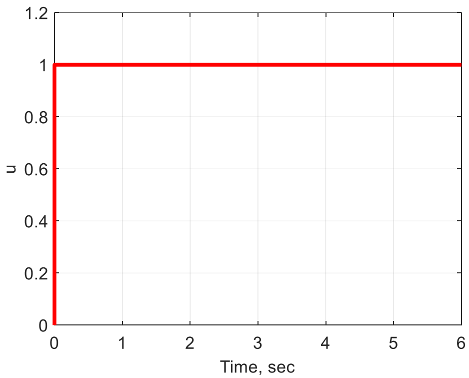
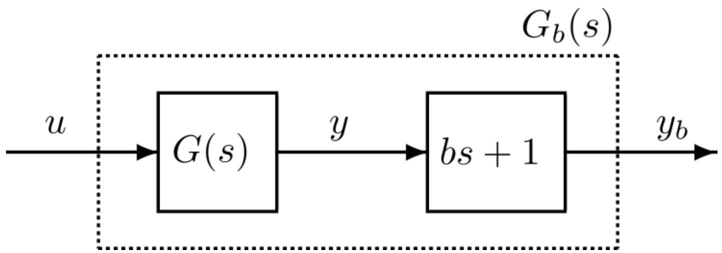


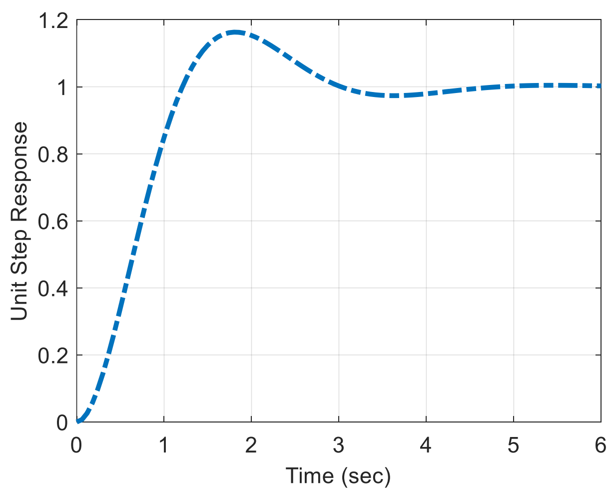


$$y_b = y + b\dot{y}$$

The zero is at:

$$z = -\frac{1}{b}$$

$$\Rightarrow y_b = y - \frac{1}{z}\dot{y}$$



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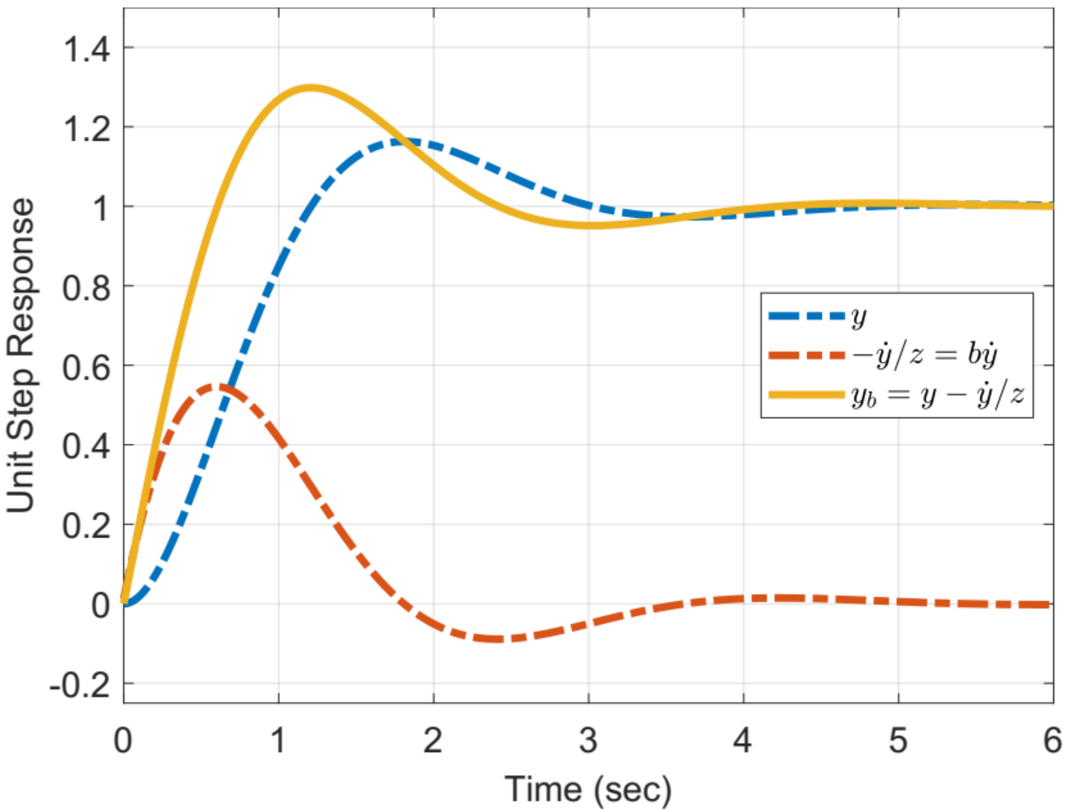


$$b = 0.5$$

$$\Rightarrow z = -\frac{1}{b} = -2$$

Effect of a LHP Zero

Consider the second-order system:

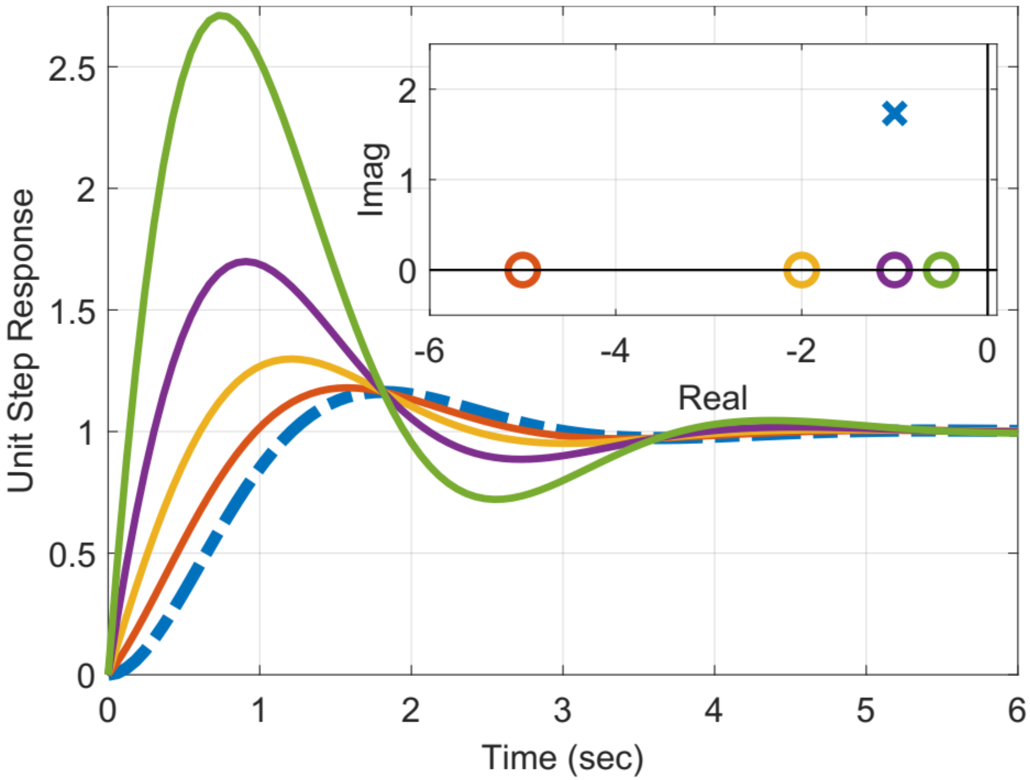
$$\ddot{y}(t) + 2\,\dot{y}(t) + 4\,y(t) = (4b)\,\dot{u}(t) + 4u(t)$$

$$G_b(s) = \frac{4b\,s+4}{s^2+2s+4} = (bs+1)\cdot G(s) \text{ where } G(s) = \frac{4}{s^2+2s+4}$$

A zero in the LHP:

- **Increases overshoot**
- **Decreases rise time**
- **No effect on settling time**

The effects are small if the zero is far in the LHP.



Effect of a RHP Zero

Consider the second-order system:

$$\ddot{y}(t) + 2\,\dot{y}(t) + 4\,y(t) = (4b)\,\dot{u}(t) + 4u(t)$$

$$G_b(s) = \frac{4b\,s+4}{s^2+2s+4} = (bs+1)\cdot G(s) \text{ where } G(s) = \frac{4}{s^2+2s+4}$$

A zero in the RHP:

- **Causes undershoot**
- **No effect on settling time**

The effects are small if the zero is far in the RHP.