ECE 486: Control Systems

Lecture 16A: Sensitivity Functions

Key Takeaways

This lectures considers a generic feedback system with plant *G(s)* and controller *K(s)*.

Two important transfer functions are:

- Sensitivity: $S(s) = \frac{1}{1+G(s)K(s)}$
- Complementary Sensitivity: $T(s) = \frac{G(s)K(s)}{1+G(s)K(s)}$

The feedback system is defined to be stable if every possible input/output transfer function is internally stable.

- This holds if and only if all zeros of 1+G(s)K(s) are in the LHP.
- The feedback system is unstable if the G(s)K(s) has a pole/zero cancellation in the CRHP.

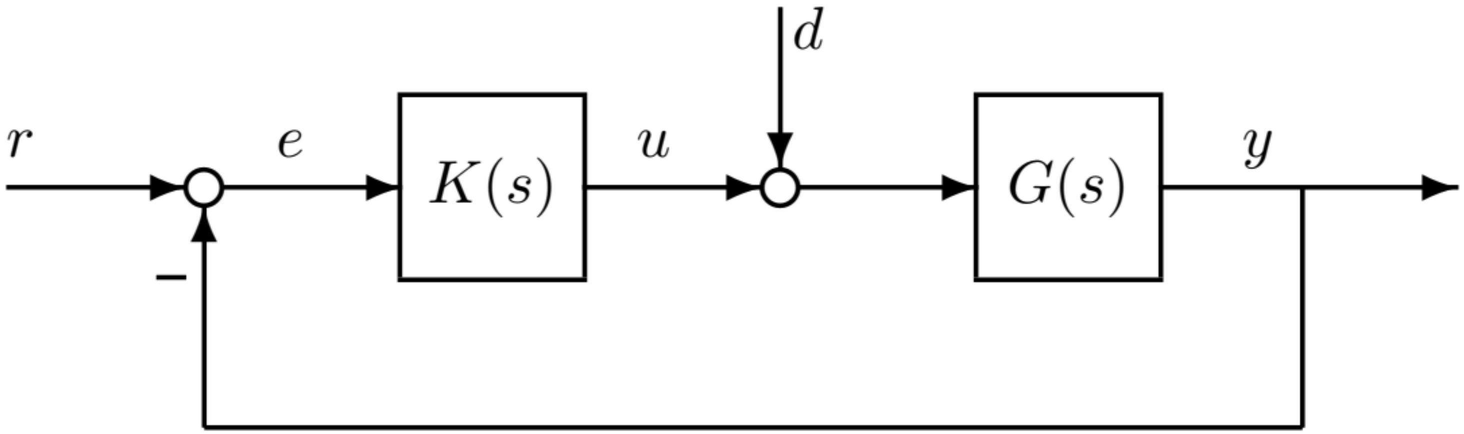
Problem 1

Consider the feedback system below.

A) What is the transfer function from disturbance *d* to output *y*? Express your answer in terms of *G(s)* and *K(s)*.

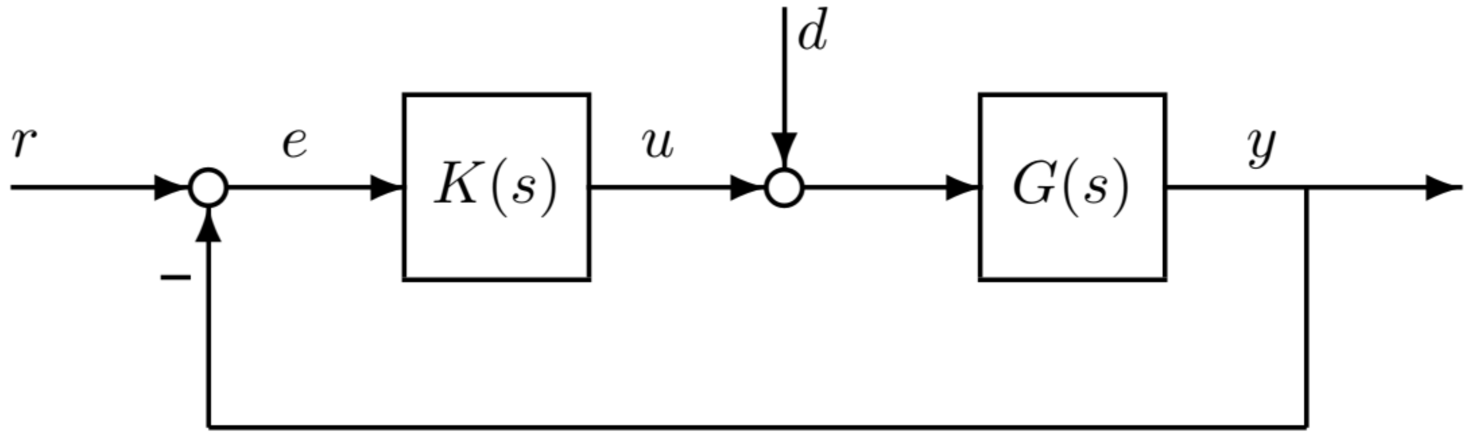
B) Is the feedback system stable if $G(s) = \frac{1}{s-2}$ and $K(s) = 5$?

C) Is the feedback system stable if $G(s) = \frac{s-1}{s+2}$ and $K(s) = \frac{5}{s-1}$?



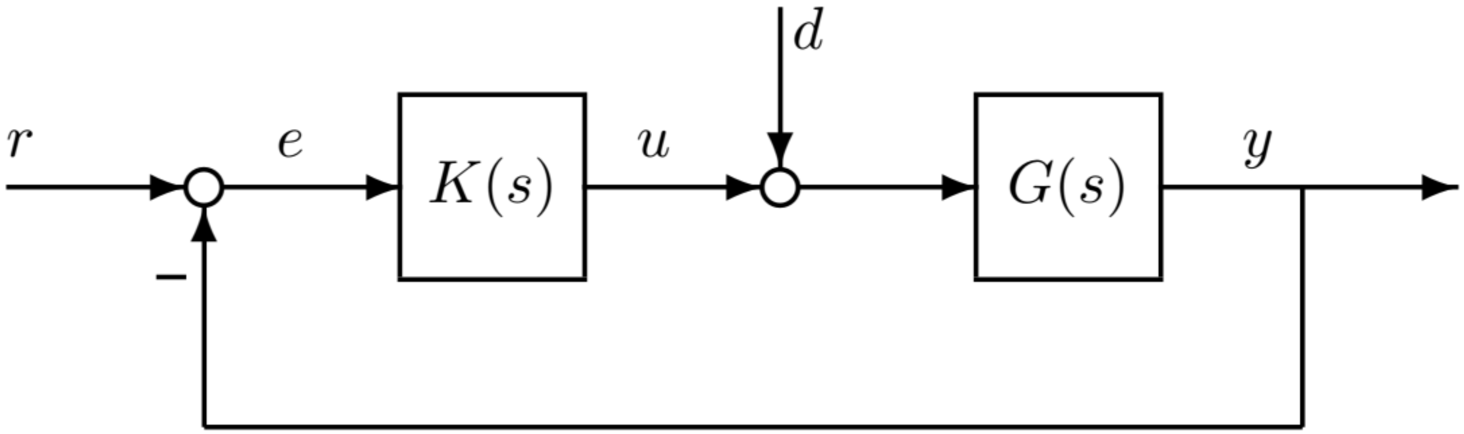
Solution 1A

A) What is the transfer function from disturbance *d* to output *y*? Express your answer in terms of *G(s)* and *K(s)*.



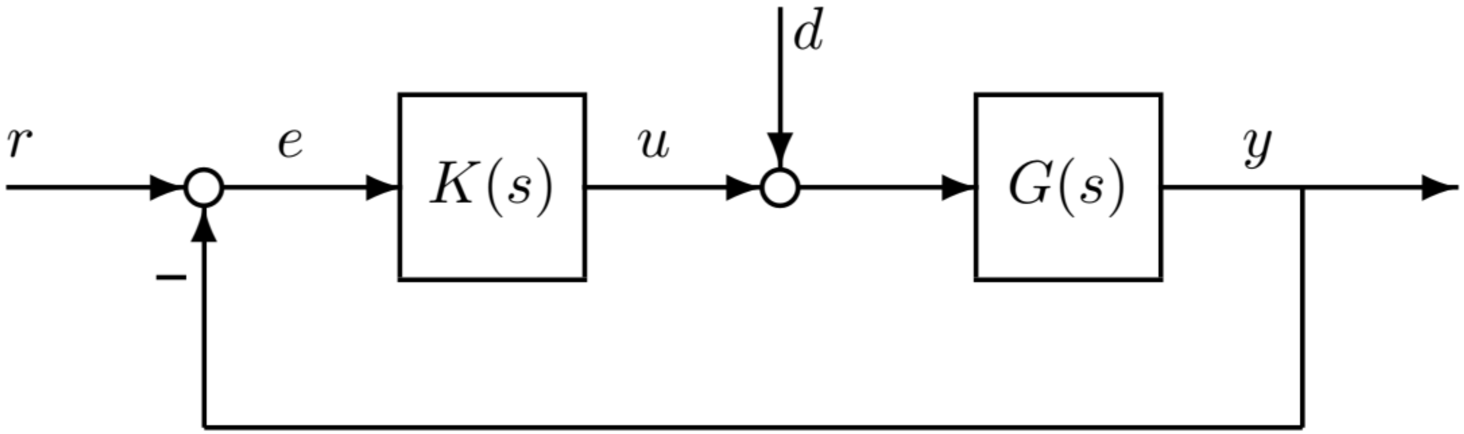
Solution 1B

B) Is the feedback system stable if $G(s) = \frac{1}{s-2}$ and $K(s) = 5$?



Solution 1C

C) Is the feedback system stable if $G(s) = \frac{s-1}{s+2}$ and $K(s) = \frac{5}{s-1}$?



Solution 1-Extra Space

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Lecture 16B: Gain Margin

Key Takeaways

This lecture discusses one safety factor called the gain margin to account for model uncertainty.

- The gain margin is the amount of allowable variation in the gain of the plant before the closed-loop becomes unstable.
- As a rule of thumb, the closed-loop should remain stable for gain variations in the range [0.5, 2] (= ±6dB).

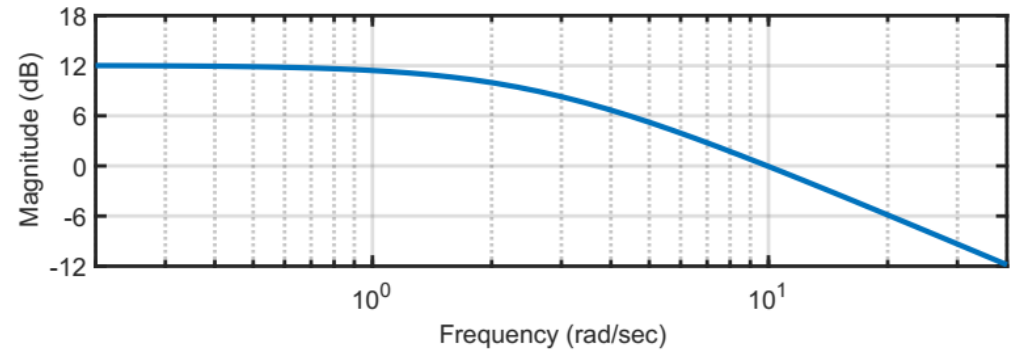
It is shown that a gain variation $g_0>0$ causes a closed-loop pole at $s = \pm j\omega_0$ if and only if $L(j\omega_0) = -1/g_0$

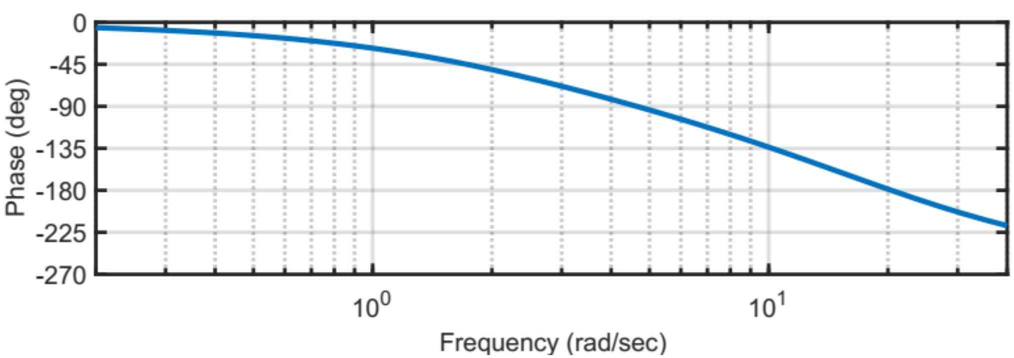
This can be used to determine gain margins from a Bode plot of the loop transfer function $L(s)$.

Problem 2

Consider a standard closed-loop system with the loop transfer function *L(s)* with Bode plot below. Assume the closed-loop is stable with the loop *L(s)*.

$$L(s) = \frac{-4s+72}{0.39s^2+8.02s+18}$$





A) What is the phase crossover frequency, ω_0?

B) What is the gain margin, g_0, of the closed-loop?

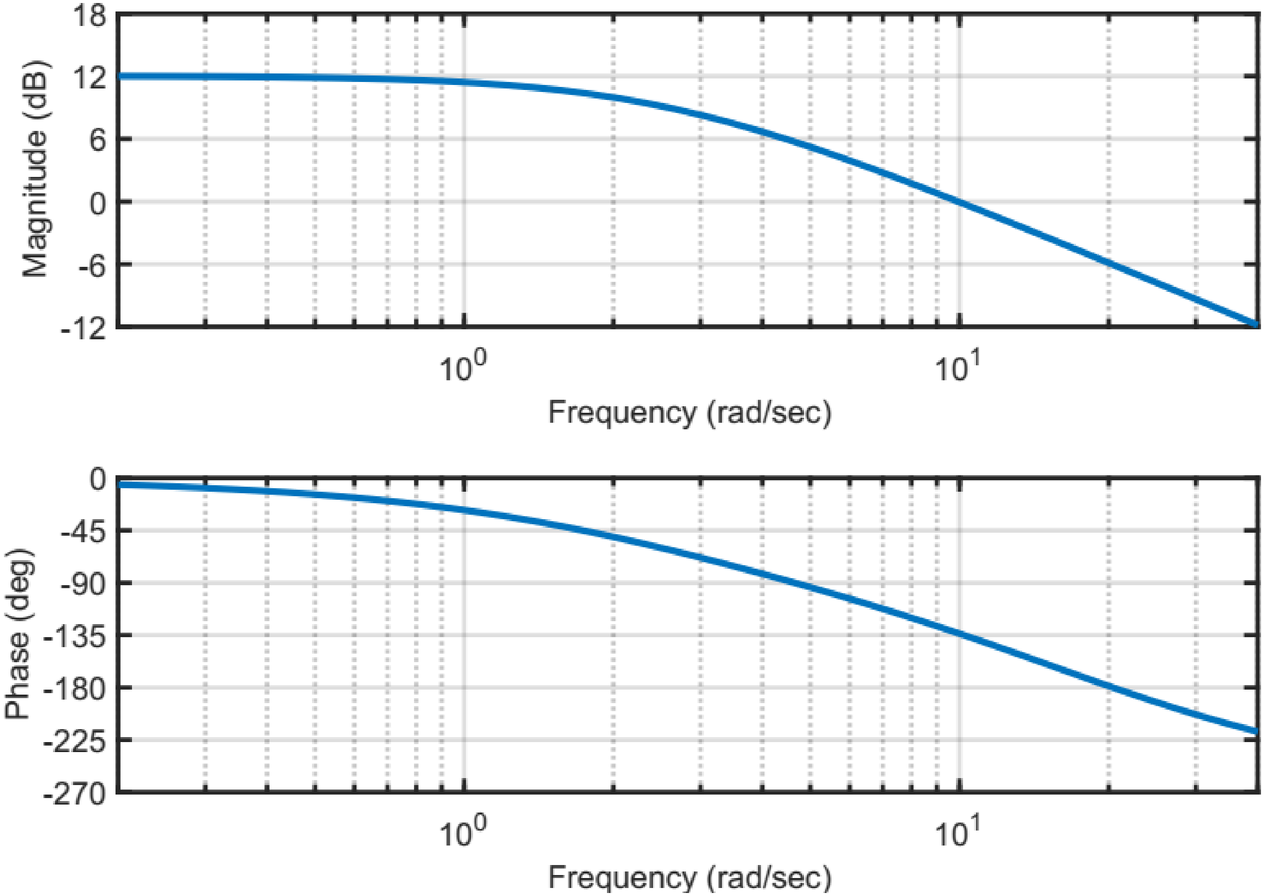
C) Is the closed-loop stable if the open-loop transfer function is *1.5L(s)*?

D) Use Matlab to verify that if the loop is $g_0 L(s)$ then the closed-loop has a pole at $j\omega_0$.

Solution 2A and 2B

A) What is the phase crossover frequency, ω_0?

B) What is the gain margin, g_0, of the closed-loop?

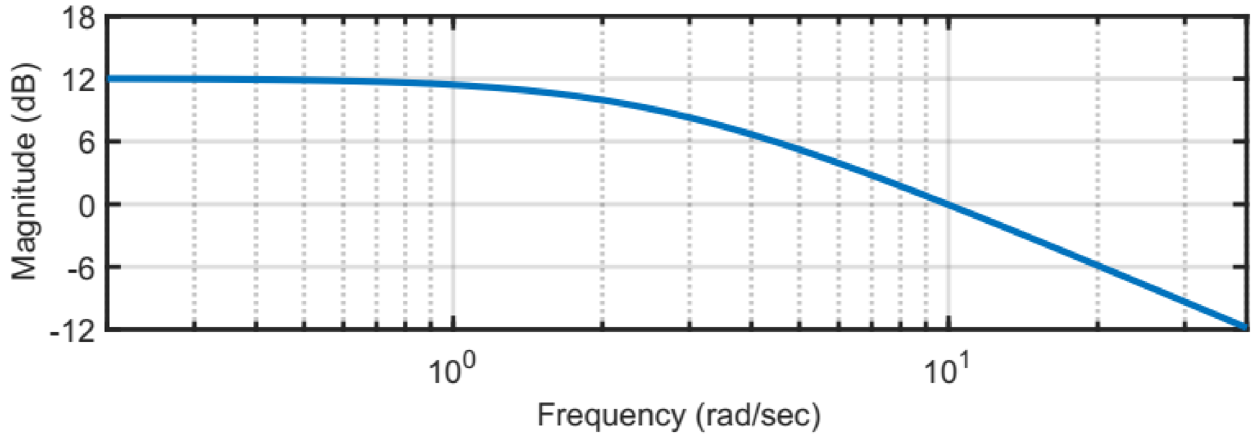


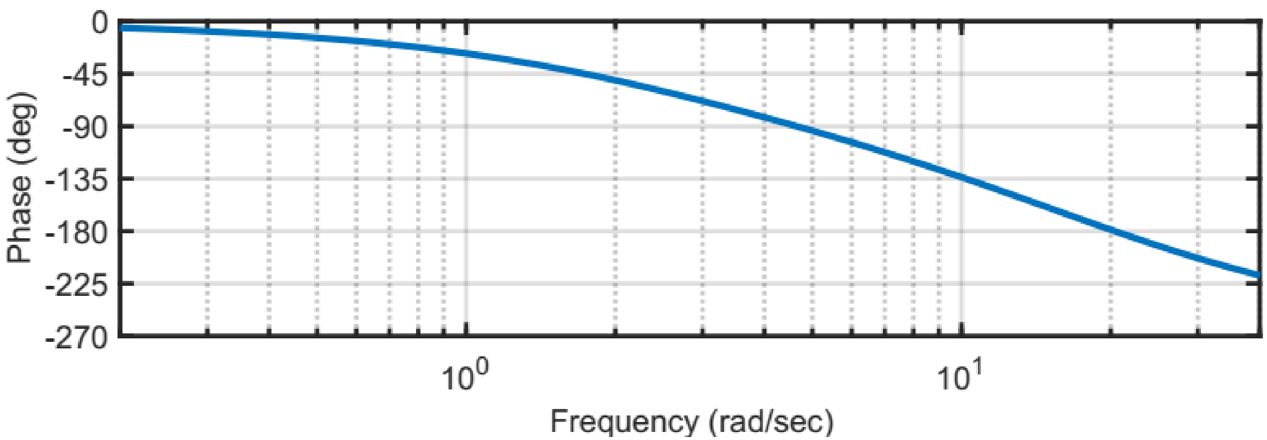
Solution 2C and 2D

C) Is the closed-loop stable if the open-loop transfer function is 1.5L(s)?

D) Use Matlab to verify that if the loop is $g_0L(s)$ then the closed-loop has a pole at $j\omega_0$.

$$L(s) = \frac{-4s+72}{0.39s^2+8.02s+18}$$





Solution 2-Extra Space

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Lecture 16C: Phase Margin

Key Takeaways

This lecture discusses another safety factor called the phase margin to account for model uncertainty and time delays.

- The phase margin is the amount of allowable variation in the phase of the plant before the closed-loop becomes unstable.
- As a rule of thumb, the closed-loop should remain stable for phase variations in the range $\pm 45^o$.

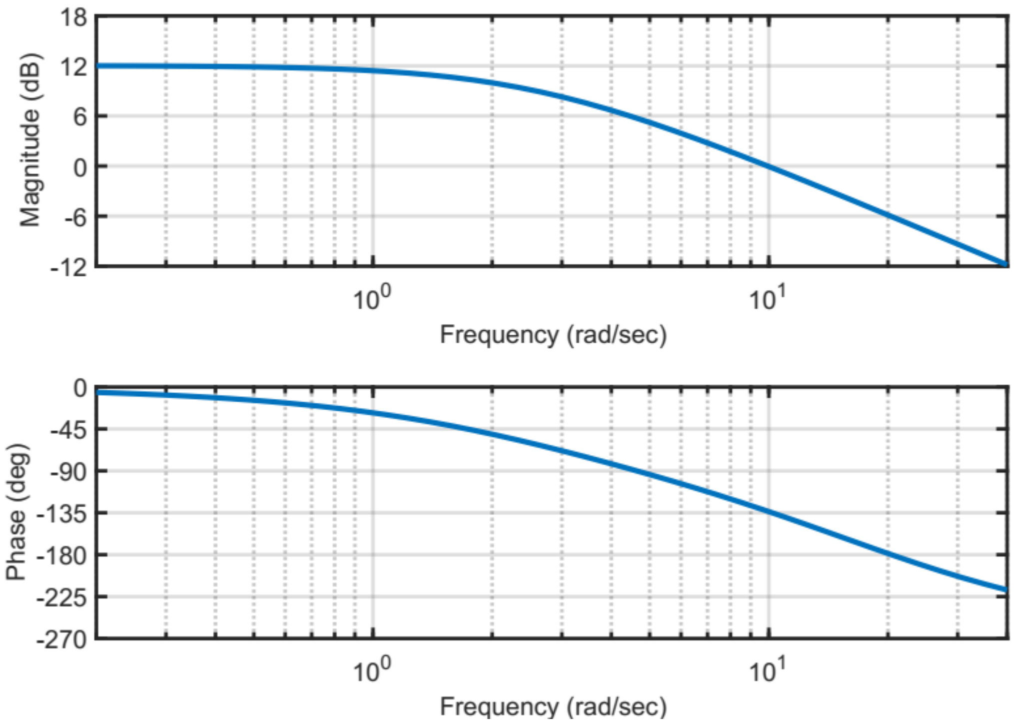
It is shown that a phase variation $\theta_0 > 0$ causes a closed-loop pole at $s = j\omega_0$ if and only if $e^{-j\theta_0} L(j\omega_0) = -1$.

This can be used to determine phase margins from a Bode plot of the loop transfer function *L(s)*.

Problem 3

Consider a standard closed-loop system with the loop transfer function *L(s)* with Bode plot below. Assume the closed-loop is stable with the loop *L(s)*.

$$L(s) = \frac{-4s+72}{0.39s^2+8.02s+18}$$



A) What is the gain crossover frequency, ω_0?

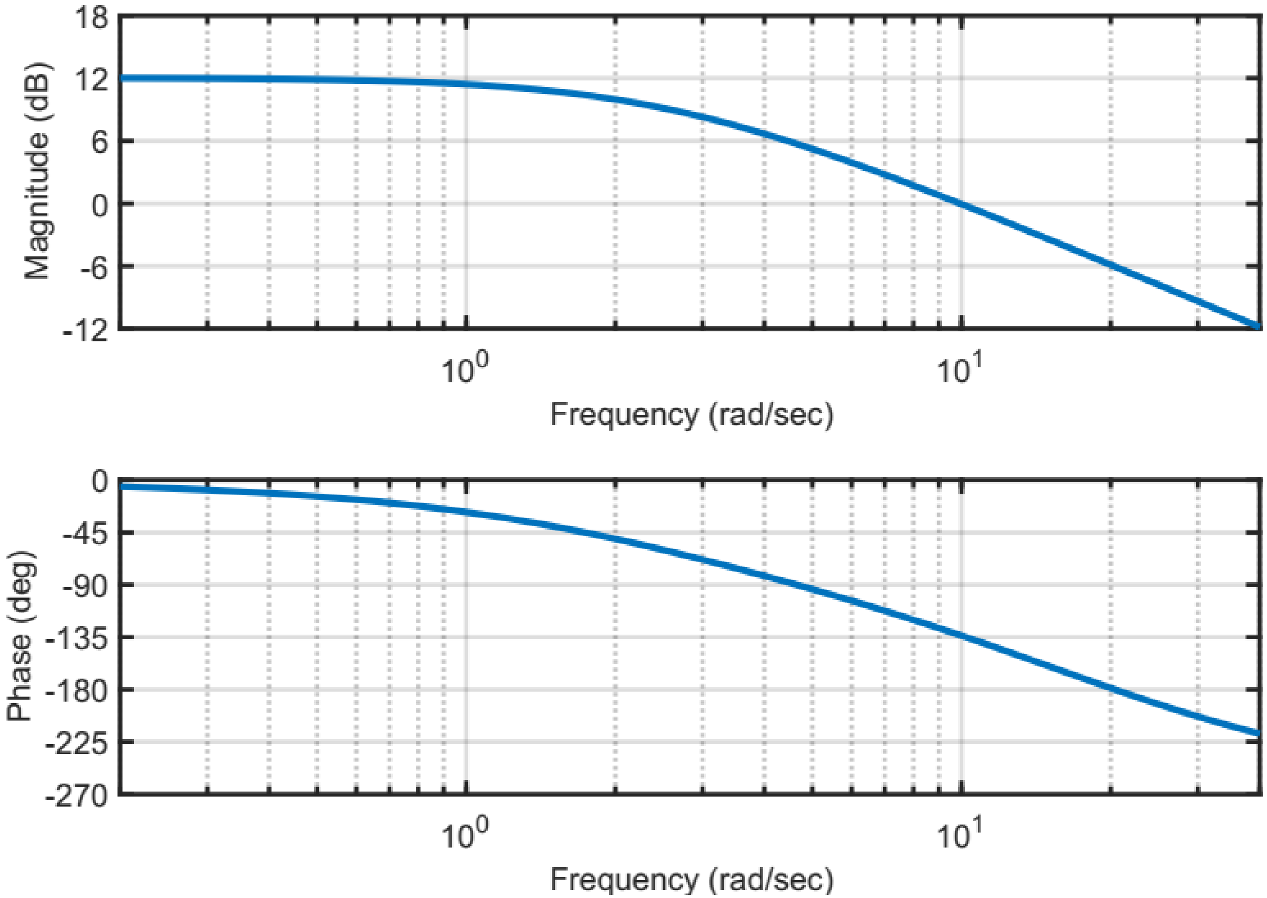
B) What is the phase margin, θ_0, of the closed-loop?

C) Use Matlab to verify that if the loop is $e^{-j\theta_0}L(s)$ then the closed-loop has a pole at $j\omega_0$.

Solution 3A and 3B

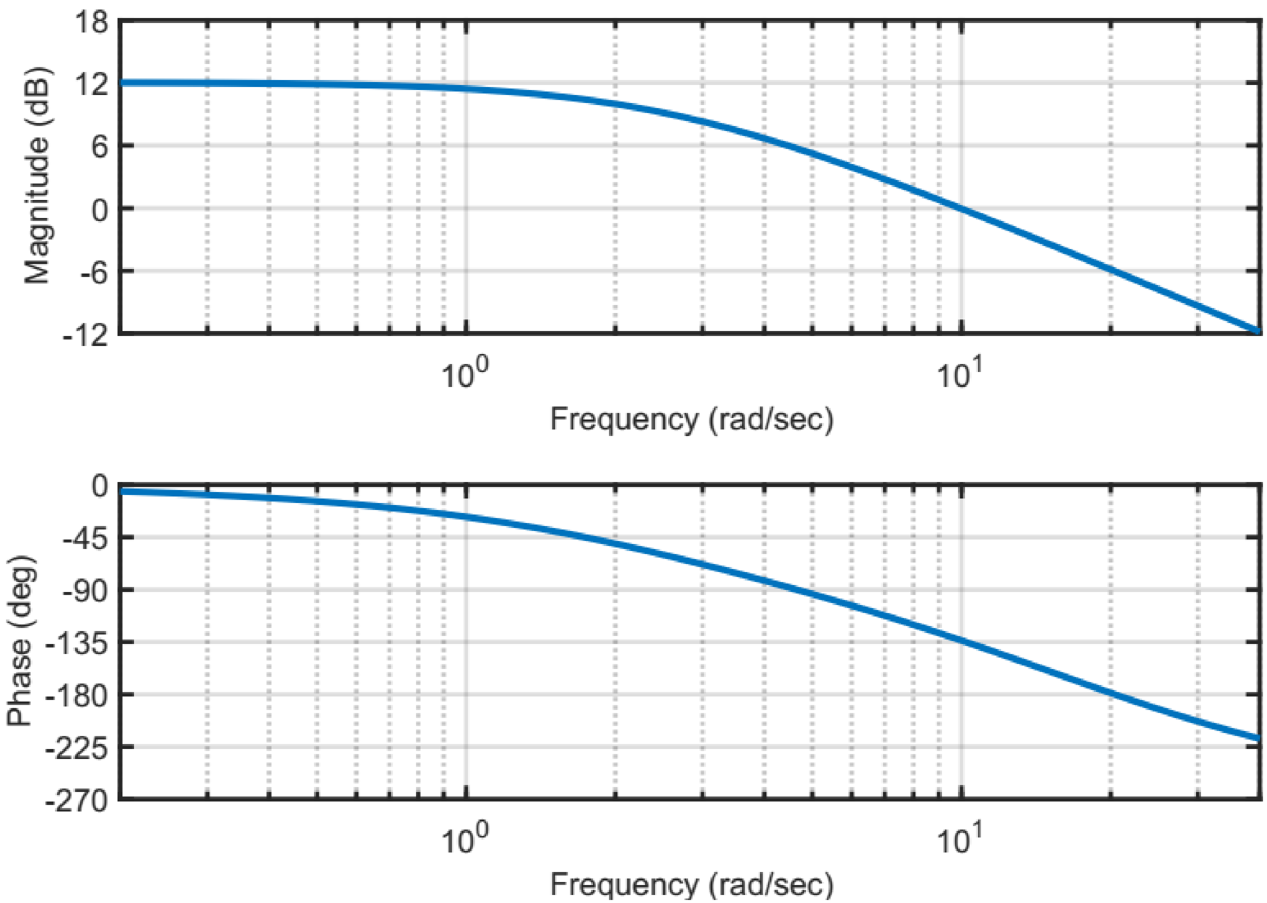
A) What is the gain crossover frequency, ω_0?

B) What is the phase margin, θ_0, of the closed-loop?



Solution 3C

C) Use Matlab to verify that if the loop is $e^{-j\theta_0}L(s)$ then the closed-loop has a pole at $j\omega_0$.



Solution 3-Extra Space