ECE 486: Control Systems

Lecture 13A: Steady-State Sinusoidal Response

Key Takeaways

The transfer function *G(s)* is used to express the solution of a stable linear system forced by a sinusoidal input.

If the input is $u(t) = \sin(\omega t)$ then the response satisfies:

$$y(t) \to |G(j\omega)|\sin(\ \omega t + \angle G(j\omega)\ ) \text{ as } t \to \infty$$

The output converges to a sinusoid at the same frequency as the input but with amplitude scaled by $|G(j\omega)|$ and phase is shifted by $\angle G(j\omega)$.

Problem 1

Consider the following first-order system and sinusoidal input:

$$-2\dot{y}(t) - y(t) = 3u(t) \qquad u(t) = 5\sin(4t + 0.1)$$

A) What is the magnitude and phase of *G(jω)*?

B) Is the steady-state response bounded? If yes, what is it?

Consider the following first-order system and sinusoidal input:

$$-2\dot{y}(t) + y(t) = 3u(t) \qquad u(t) = 5\sin(4t + 0.1)$$

C) What is the magnitude and phase of *G(jω)*?

D) Is the steady-state response bounded? If yes, what is it?

Solution 1A and 1B

Consider the following first-order system and sinusoidal input:

$$-2\dot{y}(t) - y(t) = 3u(t) \qquad\qquad u(t) = 5\sin(4t + 0.1)$$

A) What is the magnitude and phase of *G(jω)*?

B) Is the steady-state response bounded? If yes, what is it?

Solution 1C and 1D

Consider the following first-order system and sinusoidal input:

$$-2\dot{y}(t) + y(t) = 3u(t) \qquad u(t) = 5\sin(4t + 0.1)$$

C) What is the magnitude and phase of *G(jω)*?

D) Is the steady-state response bounded? If yes, what is it?

Solution 1-Extra Space

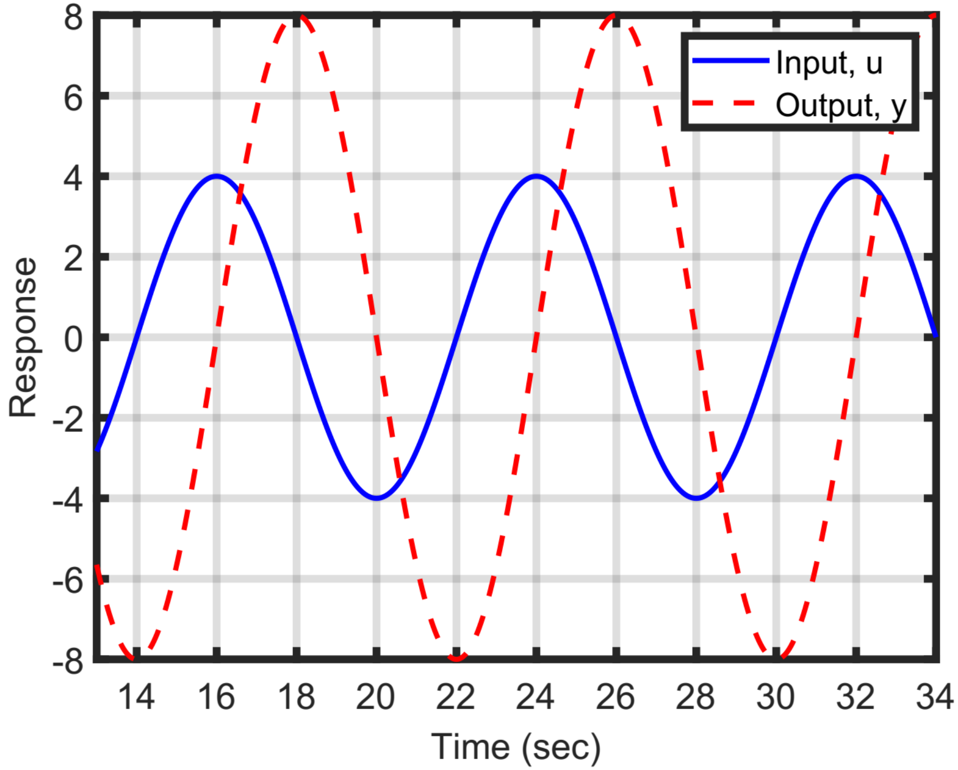
Problem 2

The figure shows the output y(t) generated by a linear system G(s) with input $u(t) = A_0 \cos(\omega_0 t)$.

A) What are the values of A_0 and ω_0 for the input signal u(t)?

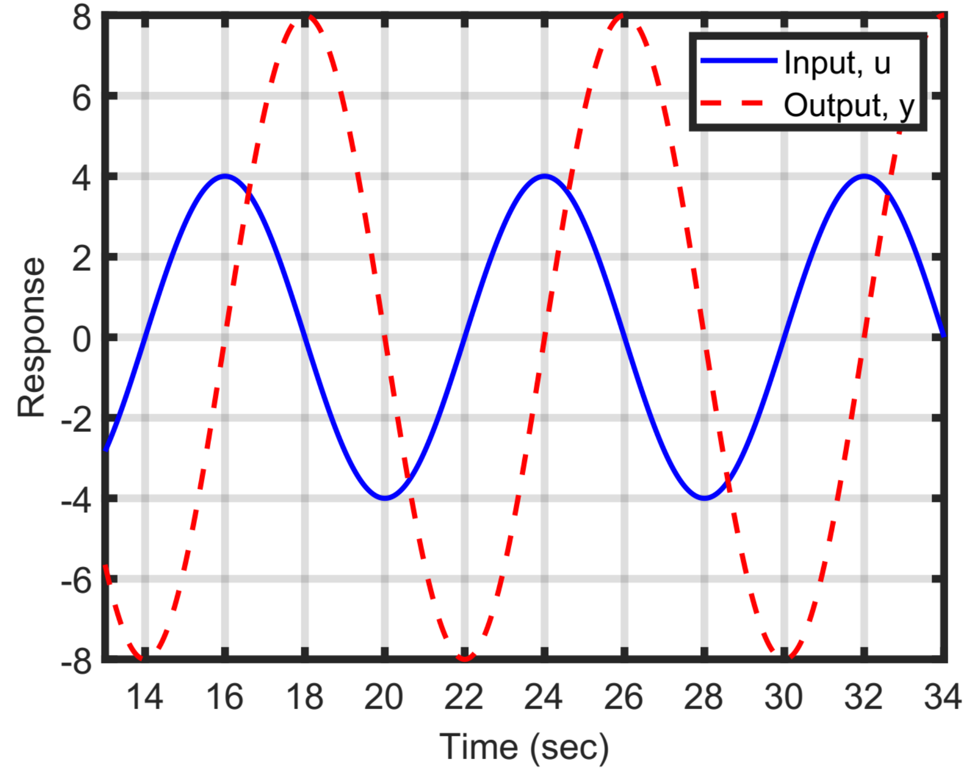
B) What is the magnitude $|G(j\omega_0)|$?

C) What is the phase $\angle G(j\omega_0)$ in degrees?



Solution 2A

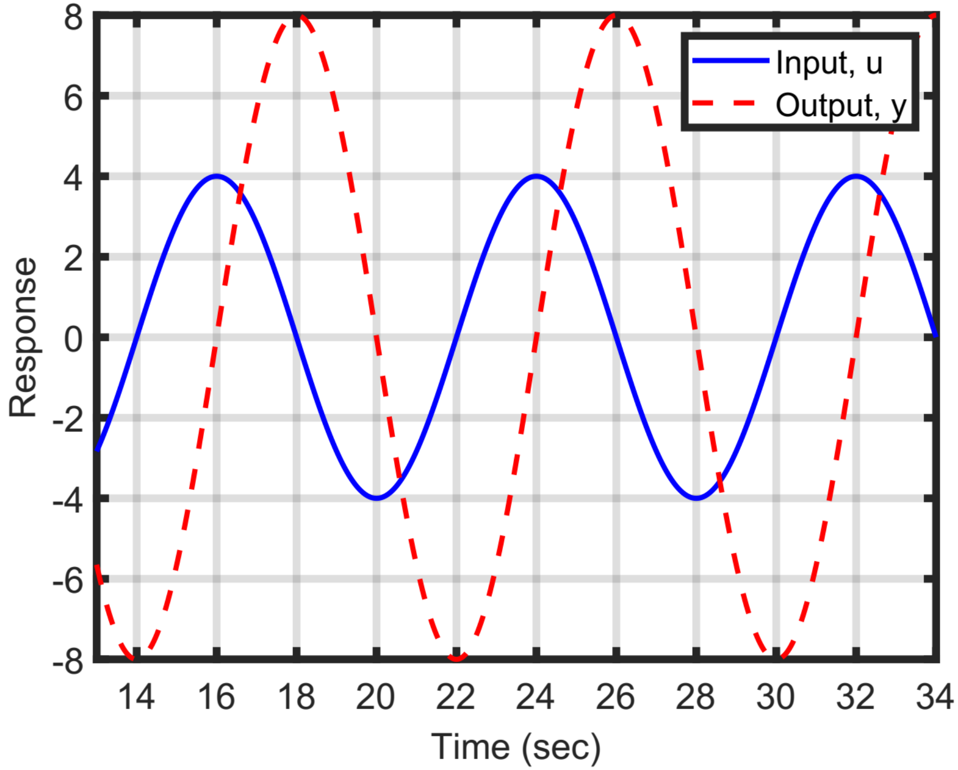
A) What are the values of A_0 and ω_0 for the input signal u(t)?



Solution 2B and 2C

B) What is the magnitude $|G(j\omega_0)|$?

C) What is the phase $\angle G(j\omega_0)$ in degrees?



Solution 2-Extra Space

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Lecture 13B: Bode Plots

Key Takeaways

A Bode plot for an LTI system *G(s)* consists of two subplots:

- Magnitude (Gain) vs. frequency and
- Phase vs. frequency.

Such plots are useful to understand the steady-state response of the system *G(s)* to sinusoids of different frequencies.

Problem 3

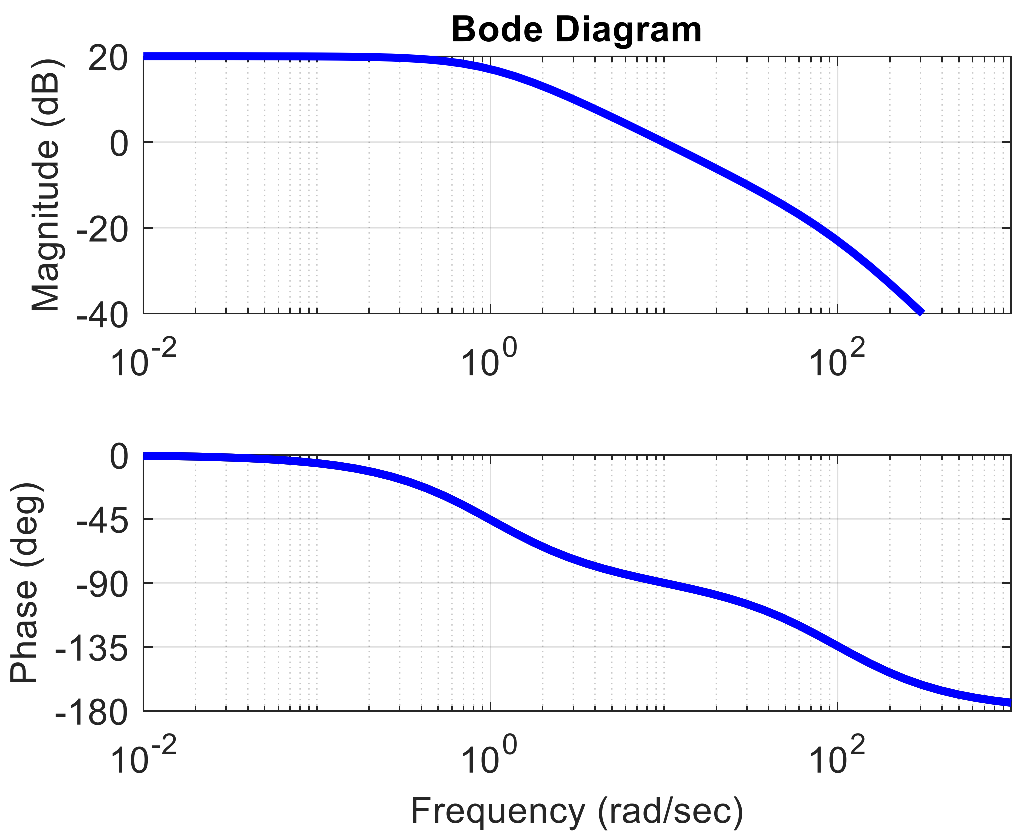
A linear system *G(s)* with input *u* and output *y* has the Bode plot shown below.

A) What is $|G(10j)|$ in dB and actual units?

B) What is $\angle G(10j)$ in degs and radians?

C) What is the output response *y(t)* in steady-state for the input *u(t) = 2 cos(10t)*?

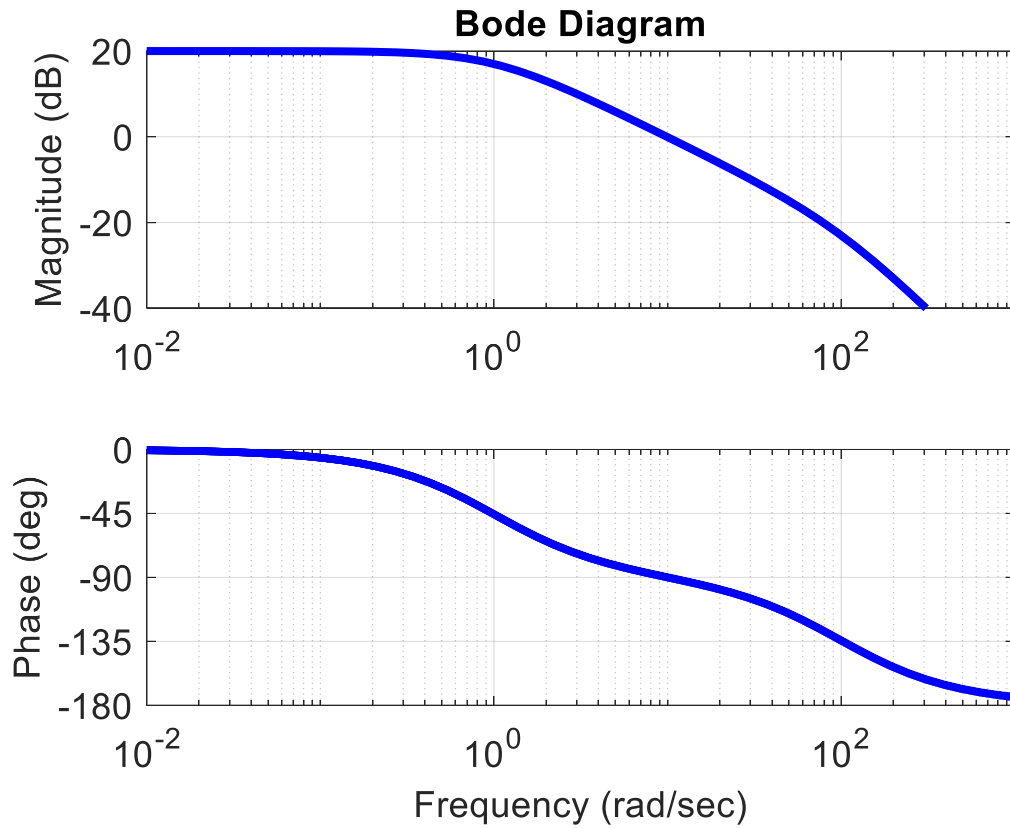
D) What is the steady-state value of y(t) if the input is a unit step u(t) = 1 for all t ≥ 0?



Solution 3A and 3B

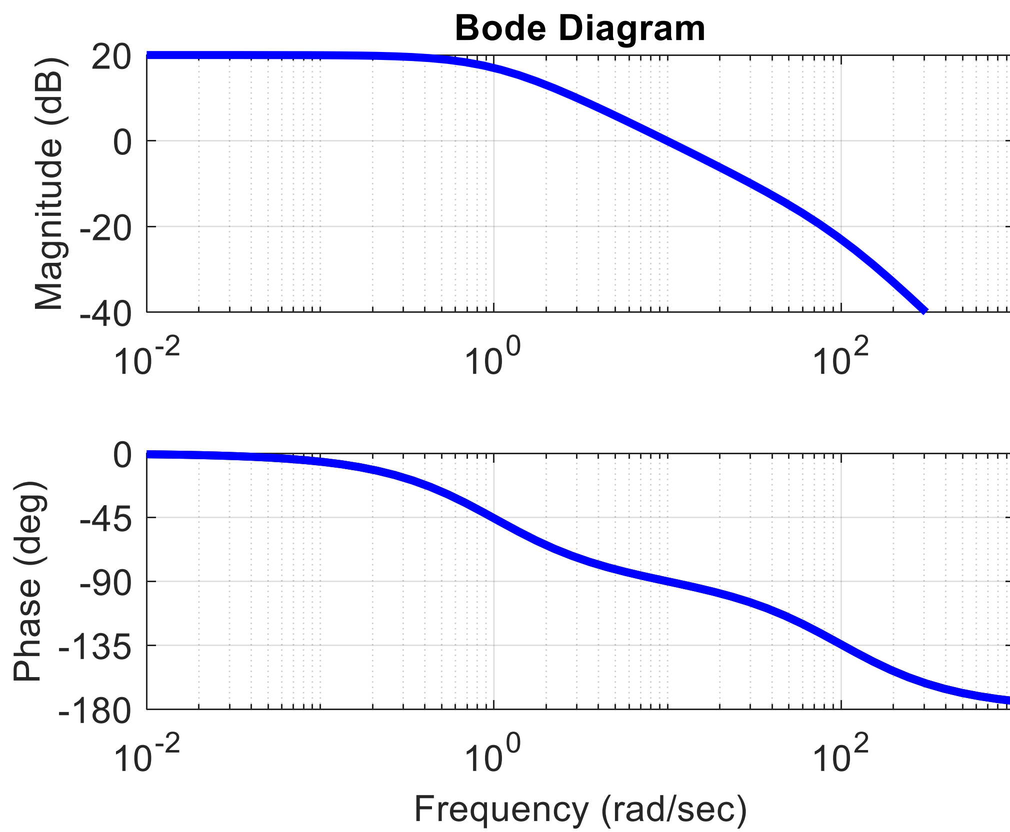
A) What is $|G(10j)|$ in dB and actual units?

B) What is $\angle G(10j)$ in degs and radians?



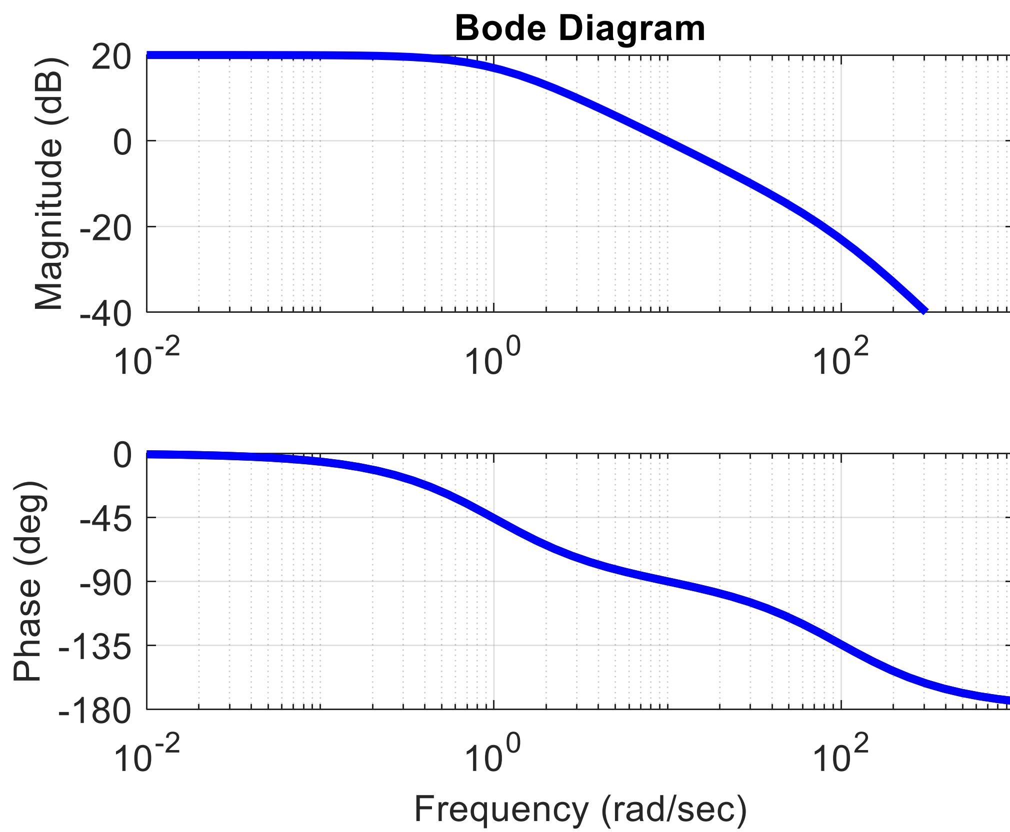
Solution 3C

C) What is the output response *y(t)* in steady-state for the input *u(t) = 2 cos(10t)*?



Solution 3D

D) What is the steady-state value of y(t) if the input is a unit step u(t) = 1 for all t ≥ 0?



Solution 3-Extra Space

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Lecture 13C: Bode Plots for First-Order Systems

Key Takeaways

This lecture focuses on Bode plots for first order systems.

The Bode plot for $G(s) = \frac{b_0}{s+a_0}$ has the following key features:

- The pole defines a corner frequency ($\omega = |a_0|$) for the system.
- The magnitude is flat at low frequencies and rolls off at −20dB per decade at high frequencies.
- The phase transitions by ±90° near the corner frequency with precise details depending on the signs of (b_0, a_0).

The Bode plot for $G(s) = \frac{s+b_0}{a_0}$ has the similar features except:

- The zero defines a corner frequency ($\omega = |b_0|$) for the system.
- The magnitude rolls up at +20dB per decade at high frequencies.

Problem 4

Sketch approximate, straight-line Bode plots for the following systems:

A) $G(s) = \frac{8}{s+4}$

B) $G(s) = \frac{16}{2s-8}$

C) $G(s) = 3s + 6$

Solution 4A

A) $G(s) = \frac{8}{s+4}$

Solution 4B

B) $G(s) = \frac{16}{2s-8}$

Solution 4C

C) $G(s) = 3s + 6$

Solution 4-Extra Space