ECE 486: Control Systems

Lecture 9A: PI Tuning for First-Order Systems
This lecture describes a method to tune PID controllers using pole placement.

For first-order systems, the approach is to:

- Use PI control and
- Select the gains to place the two closed-loop poles at desired locations.

The choice of natural frequency (time constant) is critical.
Problem 1

Consider the plant with the following transfer function:

\[ G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101} \]

The poles of this system are at \( s = -1, -10 \pm j \).

A) What is the dominant pole approximation \( G_a(s) \) for this plant?

B) Would you recommend using a PI, PD, or PID Controller?

C) Choose the controller gains so that the closed-loop with \( G_a(s) \) has poles repeated at \( s = -1 \).

D) Where are the poles for the closed-loop with your controller and the actual plant \( G(s) \)? [Use numerical tools to solve.]
A) What is the dominant pole approximation $G_d(s)$ for this plant?

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$
Solution 1B

\[ G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101} \]

B) Would you recommend using a PI, PD, or PID Controller?
C) Choose the controller gains so that the closed-loop with $G_a(s)$ has poles repeated at $s=-1$. 

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$
Solution 1D

\[ G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101} \]

D) Where are the poles for the closed-loop with your controller and the actual plant \( G(s) \)? [Use numerical tools to solve.]
Problem 2

Again consider the plant with the following transfer function:

\[ G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101} \]

The poles of this system are at \( s = -1, -10 \pm j \).

A) Rechoose your controller gains so that the closed-loop with \( G_a(s) \) has poles repeated at \( s = -2 \).

B) Where are the poles for the closed-loop with your controller and the actual plant \( G(s) \)? [Use numerical tools to solve.]

C) What is the impact, if any, of the neglected poles?
Solution 2A

\[ G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101} \]

A) Rechoose your controller gains so that the closed-loop with \( G_a(s) \) has poles repeated at \( s=-2 \).
Solution 2B and 2C

\[ G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101} \]

B) Where are the poles for the closed-loop with your controller and the actual plant \( G(s) \)? [Use numerical tools to solve.]
C) What is the impact, if any, of the neglected poles?
Solution 2-Extra Space
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Lecture 9B: PID Tuning for Second-Order Systems
This lecture describes a method to tune PID controllers using pole placement.

For second-order systems, the approach is to:

• Use PID control and

• Select the gains to place the three closed-loop poles at desired locations.

• A PI controller (without the D-term) should be used if the plant has sufficient damping.

The choice of natural frequency (time constant) is critical.
Problem 3

Consider the plant with the following transfer function:

\[ G(s) = \frac{20}{s^2 - 6s + 10} \]

A) What is the closed-loop ODE from reference \( r \) to output \( y \) if you use a PID controller? 

\[ u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \dot{e}(t) \]

B) Choose the controller gains so that the closed-loop has poles repeated at \( s=-3 \). Hint: \( (s+3)^3 = s^3 + 9s^2 + 27s + 27 \)

C) What is the impact of implementing the derivative term as \( K_d \dot{e} \) versus the rate feedback form \(-K_d \dot{y}\)?
Solution 3A

\[ G(s) = \frac{20}{s^2 - 6s + 10} \]

A) What is the closed-loop ODE from reference \( r \) to output \( y \) if you use a PID controller?

\[ u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \dot{e}(t) \]
Solution 3B

\[ G(s) = \frac{20}{s^2 - 6s + 10} \]

B) Choose the controller gains so that the closed-loop has poles repeated at \( s = -3 \). Hint: \( (s+3)^3 = s^3 + 9s^2 + 27s + 27 \)
Solution 3C

\[ G(s) = \frac{20}{s^2 - 6s + 10} \]

C) What is the impact of implementing the derivative term as \( K_d \dot{e} \) versus the rate feedback form \(-K_d \dot{y}\)?
Solution 3-Extra Space