ECE 486: Control Systems

Lecture 6A: Effect of Extra Poles & Zeros
Key Takeaways

This lecture considers the effect of extra poles and zeros on the step response.

**LHP Poles:** Increase settling time.
The effects are small if the pole is far in the LHP.

**LHP Zeros:** Increase overshoot, decrease rise time, and have no effect on settling time.
The effects are small if the zero is far in the LHP.

**RHP Zeros:** Cause undershoot but no effect on settling time.
The effects are small if the zero is far in the RHP.
Problem 1

Four systems and four unit step responses are given below. Match each system to its unit step response.

What happens if adding a pole at -20?

\[
G_A = \frac{-2s+10}{s^2+2s+5}, \quad G_C = \frac{2s+10}{s^2+2s+5}, \quad G_B = \frac{-10s+10}{s^2+2s+5}, \quad G_D = \frac{10s+10}{s^2+2s+5}
\]
Solution 1

\[ G_A = \frac{-2s+10}{s^2+2s+5}, \quad G_B = \frac{-10s+10}{s^2+2s+5}, \quad G_C = \frac{2s+10}{s^2+2s+5}, \quad G_D = \frac{10s+10}{s^2+2s+5} \]

\( s = -1 \pm 2j \) \quad \( T_S = 3 \text{sec} \) \quad All the DC Gain \( Z \)

A. \( s = +5 \) \quad RHP Undershoot \quad Far in RHP \quad (less effect)

B. \( s = +1 \)

C. \( s = -5 \) \quad LHP \quad (less effect)

D. \( s = -1 \) \quad Far in LHP \quad (less effect)

Neater to Imag. Axis \quad (more effect)

\[ G(s) = \frac{10}{s^2+2s+5} \]
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Lecture 6B: Stability
Key Takeaways

We study the properties exponential terms $e^{st}$ that appear in the free and forced response.

The lecture covers the following:

1. Response characteristics for real and complex roots
2. Time Constants
3. Internal Stability
4. Bounded-Input, Bounded-Output Stability
Problem 2

For each of the systems below:

- What are the poles? Is the system stable?

\[
G_A(s) = \frac{s-2}{s+7}
\]
\[
G_B(s) = \frac{s+2}{s-7}
\]
\[
G_C(s) = \frac{-9}{s^2+2s-8}
\]
\[
G_D(s) = \frac{5}{(s^2+4s+13)(s-5)}
\]
Solution 2A

What are the poles? Is the system stable?

\[ G_A(s) = \frac{s - 2}{s + 7} \]

\[ s = -7 \quad \text{LHP} \]

Stable

\[ T = \frac{1}{|\text{Re}(\omega)|} = \frac{1}{7} \text{ sec} \]
Solution 2B

- What are the poles? Is the system stable?

\[ G_B(s) = \frac{s + 2}{s - 7} \]

\[ s = +7 \quad \text{– RHP} \]

unstable

\[ \tau = \frac{1}{7} \text{ sec} \]
Solution 2C

- What are the poles? Is the system stable?

\[ G_C(s) = \frac{-9}{s^2 + 2s - 8} \]

- S = -4 ± 2i, \( Z = 2 \text{ sec} \)
- \( \tau_1 = \frac{1}{4} \text{ sec} \)
- \( \tau_2 = \frac{1}{2} \text{ sec} \)
- Unstable
- \( 3\tau_2 = 1.5 \text{ sec} \)
- \( 3\tau_1 = 0.75 \text{ sec} \)
Solution 2D

- What are the poles? Is the system stable?

\[ G_D(s) = \frac{5}{(s^2 + 4s + 13)(s - 5)} \]

- \( s_1 = 5 \) - Right Half Plane (RHP)
  - Unstable
  - \( \tau_1 = \frac{1}{5} \) sec

- \( s_{2,3} = -2 \pm 3j \) - Left Half Plane (LHP)
  - Slowest
  - \( \tau_{2,3} = \frac{1}{2} \) sec
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Lecture 6C: Routh-Hurwitz Criterion
Problem 3

Without a computer, determine whether or not the following polynomial have any RHP roots:

\[ s^4 + 10s^3 + 40s^2 + 20s + 1 \]

The Routh table is constructed below:

<table>
<thead>
<tr>
<th>(s^4)</th>
<th>1</th>
<th>40</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s^3)</td>
<td>10</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>(s^2)</td>
<td>38</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(s^1)</td>
<td>750/38</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(s^0)</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The polynomial has no RHP roots.