ECE 486: Control Systems

Lecture 6A: Effects of Extra Poles and Zeros
This lecture considers the effect of extra poles and zeros on the step response.

**LHP Poles:** Increase settling time.
The effects are small if the pole is far in the LHP.

**LHP Zeros:** Increase overshoot, decrease rise time, and have no effect on settling time.
The effects are small if the zero is far in the LHP.

**RHP Zeros:** Cause undershoot but no effect on settling time.
The effects are small if the zero is far in the RHP.
Problem 1

Four systems and four unit step responses are given below. Match each system to its unit step response.

What happens if adding a pole at -20?

\[ G_A = \frac{-2s+10}{s^2+2s+5} \]
\[ G_B = \frac{-10s+10}{s^2+2s+5} \]
\[ G_C = \frac{2s+10}{s^2+2s+5} \]
\[ G_D = \frac{10s+10}{s^2+2s+5} \]
Solution 1

\[ G_A = \frac{-2s+10}{s^2+2s+5}, \quad G_B = \frac{-10s+10}{s^2+2s+5}, \quad G_C = \frac{2s+10}{s^2+2s+5}, \quad G_D = \frac{10s+10}{s^2+2s+5} \]
Solution 1-Extra Space
ECE 486: Control Systems

Lecture 6B: Stability
Key Takeaways

We study the properties exponential terms $e^{st}$ that appear in the free and forced response.

The lecture covers the following:

1. Response characteristics for real and complex roots
2. Time Constants
3. Internal Stability
4. Bounded-Input, Bounded-Output Stability
Problem 2

For each of the systems below:

- What are the poles? Is the system stable?

\[
\begin{align*}
G_A(s) &= \frac{s-2}{s+7} \\
G_B(s) &= \frac{s+2}{s-7} \\
G_C(s) &= \frac{-9}{s^2+2s-8} \\
G_D(s) &= \frac{5}{(s^2+4s+13)(s-5)}
\end{align*}
\]
Solution 2A

- What are the poles? Is the system stable?

\[ G_A(s) = \frac{s - 2}{s + 7} \]
Solution 2B

- What are the poles? Is the system stable?

\[ G_B(s) = \frac{s+2}{s-7} \]
Solution 2C

- What are the poles? Is the system stable?

\[ G_C(s) = \frac{-9}{s^2 + 2s - 8} \]
Solution 2D

- What are the poles? Is the system stable?

\[ G_D(s) = \frac{5}{(s^2 + 4s + 13)(s - 5)} \]
Solution 2-Extra Space
ECE 486: Control Systems

Lecture 6C: Routh-Hurwitz Criterion
Problem 3

Without a computer, determine whether or not the following polynomial have any RHP roots:

\[ s^4 + 10s^3 + 40s^2 + 20s + 1 \]
Solution 3-Extra Space