ECE 486: Control Systems

Lecture 5A: Interconnection of Systems
Problem 1

A) Suppose $G_1(s) = \frac{3}{s+2}$ and $G_2(s) = \frac{5}{s+7}$. What is the ODE for serial connection $H(s)=G_2(s)G_1(s)$?

B) Suppose $G_1(s) = \frac{5}{s+7}$ and $G_2(s) = \frac{3}{s+2}$. What is the ODE for serial connection $H(s)=G_2(s)G_1(s)$?

C) Suppose $G_1(s) = \frac{3}{s+2}$ and $G_2(s) = \frac{5}{s+7}$. What is the ODE for parallel connection $H(s)=G_1(s) + G_2(s)$?
Problem 1

D) Consider the feedback system below with:

$$\dot{y}(t) + 5y(t) = 5u(t) \quad \text{and} \quad u(t) = 2e(t) + 4 \int_0^t e(\tau) \, d\tau$$

Obtain a model of the closed-loop from $r$ to $y$ with transfer functions, and compare your answers in Matlab using the function `feedback`.

![Diagram](image-url)
Solution 1A

A) Suppose $G_1(s) = \frac{3}{s+2}$ and $G_2(s) = \frac{5}{s+7}$. What is the ODE for serial connection $H(s) = G_2(s) \cdot G_1(s)$?

$$H = G_2 G_1 = \left( \frac{3}{s+2} \right) \left( \frac{5}{s+7} \right) = \frac{15}{(s+2)(s+7)}$$

$$= \frac{15}{s^2 + 9s + 14}$$

$$\ddot{y} + 9 \dot{y} + 14y = 15u$$
B) Suppose $G_1(s) = \frac{5}{s+7}$ and $G_2(s) = \frac{3}{s+2}$. What is the ODE for serial connection $H(s)=G_2(s)G_1(s)$?

$$G_2G_1 = G_1G_2$$

[Input and Output]
C) Suppose $G_1(s) = \frac{3}{s+2}$ and $G_2(s) = \frac{5}{s+7}$. What is the ODE for parallel connection $H(s) = G_1(s) + G_2(s)$?

\[ H = G_1 + G_2 = \frac{3}{s+2} + \frac{5}{s+7} \]

\[ = \frac{3(s+7) + 5(s+2)}{(s+2)(s+7)} = \frac{8s + 24}{s^2 + 9s + 14} \]

\[ y'' + 9y' + 14y = 8u + 24u \]
Consider the feedback system below with:

\[ \dot{y}(t) + 5y(t) = 5u(t) \quad \text{and} \quad u(t) = 2e(t) + 4 \int_0^t e(\tau) \, d\tau \]

B) Obtain a model of the closed-loop from \( r \) to \( y \) with transfer functions.

\[
T_{r \rightarrow y}(s) = \frac{G(s) K(s)}{1 + G(s) K(s)} = \frac{(5s + 5)(2s + 4)/s}{1 + (5s + 5)(2s + 4)/s} = \frac{10s + 20}{s^2 + 15s + 20} = \frac{s^2 + 5s}{s^2 + 15s + 20}
\]
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Lecture 5B: Block Diagrams
Problem 2

A) Draw a block diagram for \( G_1(s) = \frac{7}{s^2 + 2s - 3} \) using integrator, summation, and gain blocks.

B) Draw a block diagram for \( G_1(s) = \frac{5s + 6}{s^2 + 2s - 3} \) using integrator, summation, and gain blocks.
A) Draw a block diagram for $G_1(s) = \frac{7}{s^2 + 2s - 3}$ using integrator, summation, and gain blocks.

$\dot{y} + 2\dot{y} - 3y = 7u \rightarrow \dot{y} = 7u - 2\dot{y} + 3y$
Solution 2B

B) Draw a block diagram for $G_1(s) = \frac{5s+6}{s^2+2s-3}$ using integrator, summation, and gain blocks.
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Lecture 5C: State-Space Models
Solution 3

Find a state-space representation for:

\[ y^{[3]}(t) + 2\ddot{y}(t) - 4\dot{y}(t) + 10y(t) = -3\dot{u}(t) + 6u(t) \]

\[
X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
\]

\[
\frac{dx}{dt} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -2 & -1 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u
\]

\[
y = \begin{bmatrix} 6 & -3 & 0 \end{bmatrix} X
\]

\[
G(s) = \frac{-3s + 6}{s^3 + 2s^2 - 4s + 10}
\]