ECE 486: Control Systems

Lecture 5A: Interconnections
Transfer functions can be used to derive models for interconnections of LTI systems.

This lecture covers two specific examples:

• The parallel connection of $G_1(s)$ and $G_2(s)$ is given by $H(s) = G_1(s) + G_2(s)$.

• The serial connection of $G_1(s)$ and $G_2(s)$ is given by $H(s) = G_1(s) \cdot G_2(s)$.

• The negative feedback interconnection of $G_1(s)$ and $G_2(s)$ is given by $H(s) = \frac{G_1(s)}{1+G_1(s) \cdot G_2(s)}$. 
Problem 1

A) Suppose \( G_1(s) = \frac{3}{s+2} \) and \( G_2(s) = \frac{5}{s+7} \). What is the ODE for serial connection \( H(s)=G_2(s) \cdot G_1(s) \)?

B) Suppose \( G_1(s) = \frac{5}{s+7} \) and \( G_2(s) = \frac{3}{s+2} \). What is the ODE for serial connection \( H(s)=G_2(s) \cdot G_1(s) \)?

C) Suppose \( G_1(s) = \frac{3}{s+2} \) and \( G_2(s) = \frac{5}{s+7} \). What is the ODE for parallel connection \( H(s)=G_1(s) + G_2(s) \)?
Problem 1

D) Consider the feedback system below with:
\[ \dot{y}(t) + 5y(t) = 5u(t) \] and \[ u(t) = 2e(t) + 4 \int_0^t e(\tau) \, d\tau \]

Obtain a model of the closed-loop from \( r \) to \( y \) with transfer functions, and compare your answers in Matlab using the function `feedback`.
A) Suppose $G_1(s) = \frac{3}{s+2}$ and $G_2(s) = \frac{5}{s+7}$. What is the ODE for serial connection $H(s)=G_2(s) G_1(s)$?
B) Suppose $G_1(s) = \frac{5}{s+7}$ and $G_2(s) = \frac{3}{s+2}$. What is the ODE for serial connection $H(s)=G_2(s) \cdot G_1(s)$?
C) Suppose $G_1(s) = \frac{3}{s+2}$ and $G_2(s) = \frac{5}{s+7}$. What is the ODE for parallel connection $H(s) = G_1(s) + G_2(s)$?
Consider the feedback system below with:

\[ \dot{y}(t) + 5y(t) = 5u(t) \text{ and } u(t) = 2e(t) + 4 \int_0^t e(\tau) \, d\tau \]

D) Obtain a model of the closed-loop from \( r \) to \( y \) and compare with your answers in Matlab using the function \texttt{feedback}.
Solution 1 – Extra Space
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Lecture 5B: Block Diagrams for Linear ODEs
Key Takeaways

• This lecture describes a method to construct block diagrams for linear ODEs with constant coefficients.
• The diagrams are constructed from blocks for:
  • integration,
  • addition/subtraction, and
  • multiplication by a gain
• These diagrams will be used later for:
  • Numerical integration of ODEs using a tool call Simulink
  • Developing state-space models. These provide an alternative to the ODE/TF models that we are using as a starting point.
A) Draw a block diagram for $G_1(s) = \frac{7}{s^2+2s-3}$ using integrator, summation, and gain blocks.

B) Draw a block diagram for $G_1(s) = \frac{5s+6}{s^2+2s-3}$ using integrator, summation, and gain blocks.
A) Draw a block diagram for \( G_1(s) = \frac{7}{s^2 + 2s - 3} \) using integrator, summation, and gain blocks.
Solution 2B

B) Draw a block diagram for $G_1(s) = \frac{5s+6}{s^2+2s-3}$ using integrator, summation, and gain blocks.
Solution 2 – Extra Space
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Lecture 5C: State-Space Models
Key Takeaways

This lecture introduces linear state-space models.

An $n^{\text{th}}$-order linear state-space model expresses the dynamics as a first-order, vector differential equation. It is possible to express as an equivalent $n^{\text{th}}$-order linear ODE.

State-space models have several uses:

• There are different tools for analysis and design of feedback systems based on state-space models.
• They can be used to approximate a nonlinear model by a related linear model.
Problem 3

Find a state-space representation for:

\[ y^{[3]}(t) + 2\ddot{y}(t) - 4\dot{y}(t) + 10y(t) = -3u(t) + 6u(t) \]
Solution 3

Find a state-space representation for:

\[ y^{[3]}(t) + 2\ddot{y}(t) - 4\dot{y}(t) + 10y(t) = -3\dot{u}(t) + 6u(t) \]