ECE 486: Control Systems

Lecture 5B: Block Diagrams
Key Takeaways

• This lecture describes a method to construct block diagrams for linear ODEs with constant coefficients.
• The diagrams are constructed from blocks for:
  • integration,
  • addition/subtraction, and
  • multiplication by a gain
• These diagrams will be used later for:
  • Numerical integration of ODEs using a tool call Simulink
  • Developing state-space models. These provide an alternative to the ODE/TF models that we are using as a starting point.
The integrator is the basic building block for graphical representations of ODEs.

Consider the 1st order system with input $u$ and output $y$:

\[
\begin{align*}
\dot{y}(t) &= u(t) \\
y(0) &= y_0
\end{align*}
\]

Obtain $y$ by integrating $u$ from the given initial condition:

\[
y(t) = y_0 + \int_0^t u(\alpha) \, d\alpha
\]
Second-order system with no input derivatives:

\[ a_2 \ddot{y}(t) + a_1 \dot{y}(t) + a_0 y(t) = b_0 u(t) \]

ICs: \( y(0) = y_0; \ \dot{y}(0) = \dot{y}_0 \)

First draw two integrators to go from \( \ddot{y}(t) \) to \( y(t) \):

An \( n^{th} \)-order system would require \( n \) integrators
Second-order system with no input derivatives:

\[ a_2 \ddot{y}(t) + a_1 \dot{y}(t) + a_0 y(t) = b_0 u(t) \]

ICs: \( y(0) = y_0; \ \dot{y}(0) = \dot{y}_0 \)

Next, solve for the highest derivative output term:

\[ a_2 \ddot{y}(t) = b_0 u(t) - a_1 \dot{y}(t) - a_0 y(t) \]
Block Diagram: No Derivatives of Input

Second-order system with no input derivatives:

\[ a_2 \ddot{y}(t) + a_1 \dot{y}(t) + a_0 y(t) = b_0 u(t) \]

ICs: \( y(0) = y_0; \ \dot{y}(0) = \dot{y}_0 \)

Next, solve for the highest derivative output term:

\[ a_2 \ddot{y}(t) = b_0 u(t) - a_1 \dot{y}(t) - a_0 y(t) \]
Second-order system with input derivatives:

\[
a_2 \ddot{y}(t) + a_1 \dot{y}(t) + a_0 y(t) = b_2 \ddot{u}(t) + b_1 \dot{u}(t) + b_0 u(t)
\]

ICs: \( y(0) = y_0; \dot{y}(0) = \dot{y}_0 \)

We could modify our previous block diagram to add derivative blocks (\( \frac{d}{dt} \)) to compute \( \dot{u} \) and \( \ddot{u} \) from \( u \).

**Computational Issue:** Differentiating tends to increase numerical errors.

**Theoretical Issue:** This requires the input signal \( u \) to be twice differentiable.
Second-order system with input derivatives:

\[ a_2 \ddot{y}(t) + a_1 \dot{y}(t) + a_0 y(t) = b_2 \ddot{u}(t) + b_1 \dot{u}(t) + b_0 u(t) \]

ICs: \( y(0) = y_0 \); \( \dot{y}(0) = \dot{y}_0 \)

Instead, represent the system as a serial interconnection:

\[
H(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0}
\]

\[
G_{u \rightarrow w}(s) = \frac{1}{a_2 s^2 + a_1 s + a_0} \quad \text{and} \quad G_{w \rightarrow y}(s) = \frac{b_2 s^2 + b_1 s + b_0}{1}
\]
Block Diagram: With Input Derivatives

Second-order system with input derivatives:

\[ a_2 \ddot{y}(t) + a_1 \dot{y}(t) + a_0 y(t) = b_2 \ddot{u}(t) + b_1 \dot{u}(t) + b_0 u(t) \]

ICs: \( y(0) = y_0; \ \dot{y}(0) = \dot{y}_0 \)

Instead, represent the system as a serial interconnection:

\[ a_2 \ddot{w}(t) + a_1 \dot{w}(t) + a_0 w(t) = u(t) \]

\[ y(t) = b_2 \ddot{w}(t) + b_1 \dot{w}(t) + b_0 w(t) \]

\[ G_{u \to w}(s) = \frac{1}{a_2 s^2 + a_1 s + a_0} \]

and \( G_{w \to y}(s) = \frac{b_2 s^2 + b_1 s + b_0}{1} \)
Second-order system with input derivatives:

\[ a_2 \ddot{y}(t) + a_1 \dot{y}(t) + a_0 y(t) = b_2 \ddot{u}(t) + b_1 \dot{u}(t) + b_0 u(t) \]

ICs: \( y(0) = y_0; \ \dot{y}(0) = \dot{y}_0 \)