ECE 486: Control Systems

Lecture 4A: Time Domain Performance
Key Takeaways

This lecture defines important performance characteristics for a system in terms of its step response.

The performance characteristics include:

- Stability
- Final Value
- Settling Time
- Overshoot
- Rise Time
- Undershoot
Problem 1

Several unit step responses are shown below. For each:

- Is the system stable?
- If the response is stable: What is the final value, settling time, overshoot, rise time? Does the response have undershoot?
Solution 1A

- Is the system stable?
- If the response is stable: What is the final value, settling time, overshoot, rise time? Does the response have undershoot?
Solution 1B

- Is the system stable?
- If the response is stable: What is the final value, settling time, overshoot, rise time? Does the response have undershoot?
Solution 1C

• Is the system stable?
• If the response is stable: What is the final value, settling time, overshoot, rise time? Does the response have undershoot?
Solution 1D

- Is the system stable?
- If the response is stable: What is the final value, settling time, overshoot, rise time? Does the response have undershoot?
Solution 1-Extra Space
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Lecture 4B: First-Order Step Response
Key Takeaways

This lecture covers the step response for first-order systems.

The step response of a *stable*, first-order system.

1. Converges to the final value with neither overshoot nor oscillations.
2. Has a settling time of three time constants.
Problem 2

A) Roughly sketch the response for the following:

\[ \dot{y}(t) + 2y(t) = 4u(t) \]

with \( y(0) = 0 \) and \( u(t) = 3 \) for all \( t \geq 0 \)

B) Roughly sketch the response for the following

\[ \dot{y}(t) - 3y(t) = 2u(t) \]

with \( y(0) = 0 \) and \( u(t) = 1 \) for all \( t \geq 0 \)
A) Roughly sketch the response for the following:

\[ \dot{y}(t) + 2y(t) = 4u(t) \]

with \( y(0) = 0 \) and \( u(t) = 3 \) for all \( t \geq 0 \)
B) Roughly sketch the response for the following

\[ \dot{y}(t) - 3y(t) = 2u(t) \]

with \( y(0) = 0 \) and \( u(t) = 1 \) for all \( t \geq 0 \)
Solution 2-Extra Space
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Lecture 4C: Second-Order Step Response
Key Takeaways

This lecture covers the step response for second-order systems.

The step response of a *stable*, second-order system.

1. Is characterized by the natural frequency and damping ratio of the system
2. Has overshoot and oscillations if the system is underdamped.
Problem 3

Each of the second-order systems below is stable*

For each system:

• What is the natural frequency and damping ratio?
• Is the system under, over, or critically damped?
• Roughly sketch the unit step response noting the final time, settling time, and overshoot (if underdamped).

\[ G_A(s) = \frac{20}{s^2 + 2s + 10} \quad \quad G_B(s) = \frac{20}{s^2 + 11s + 10} \]

*Recall that \( s^2 + a_1s + a_0 = 0 \) has all poles in the LHP if and only if \( a_1 > 0 \) and \( a_0 > 0 \).
Solution 3A

• What is the natural frequency and damping ratio?
• Is the system under, over, or critically damped?
• Roughly sketch the unit step response noting the final time, settling time, and overshoot (if underdamped).

\[ G_A(s) = \frac{20}{s^2 + 2s + 10} \]
Solution 3B

• What is the natural frequency and damping ratio?
• Is the system under, over, or critically damped?
• Roughly sketch the unit step response noting the final time, settling time, and overshoot (if underdamped).

\[ G_B(s) = \frac{20}{s^2 + 11s + 10} \]
Solution 3-Extra Space