ECE 486: Control Systems

Lecture 18D: Margins from Nyquist Plots

Key Takeaways

This lecture discusses how to read gain/phase margins from the Nyquist plots.

- The Nyquist curve of *L(s)* must encircle the critical *s=-1* point the correct number of times for closed-loop stability.
- Gain and phase variations can cause the Nyquist curve to have an “incorrect" number of encirclements (unstable closed-loop).
- The disk margin measures the minimum distance from the Nyquist curve of *L(s)* to the *s=-1* point: $d_{min} = \min_{\omega} |1 + L(j\omega)|$

Margins on a Nyquist Plot

Consider the feedback system with:

$$G(s) = \frac{1}{s^3+2s^2+3s-1} \text{ and } K(s) = 2.$$

$$\Rightarrow L(s) = G(s)K(s) = \frac{2}{s^3+2s^2+3s-1}.$$

Poles of open loop *L(s)* are:

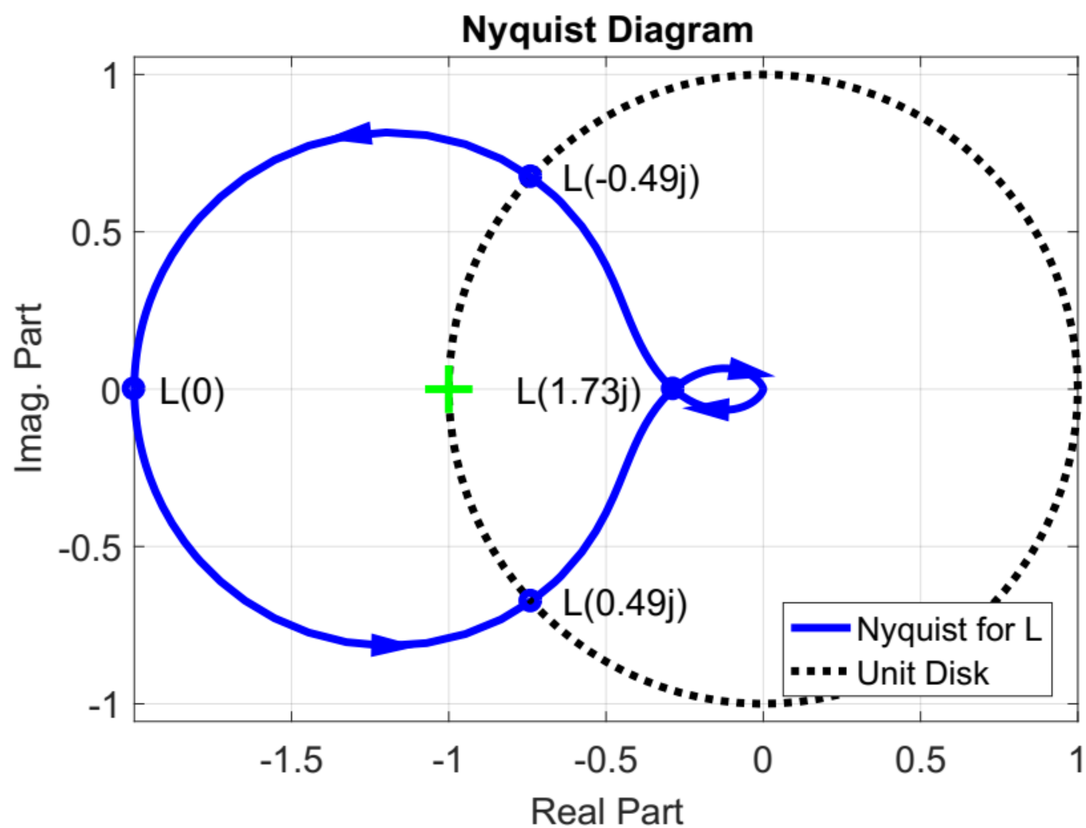
$$s_{1,2} = -1.14 \pm 1.53j$$
$$s_3 = 0.28$$

Nyquist Theorem

- $P_{OL} = +1$
- $N_{CCW} = +1$

$\rightarrow$ $P_{CL} = P_{OL} - N_{CCW.} = 0.$

Nominal closed-loop is stable.



Margins on a Nyquist Plot

Consider the feedback system with:

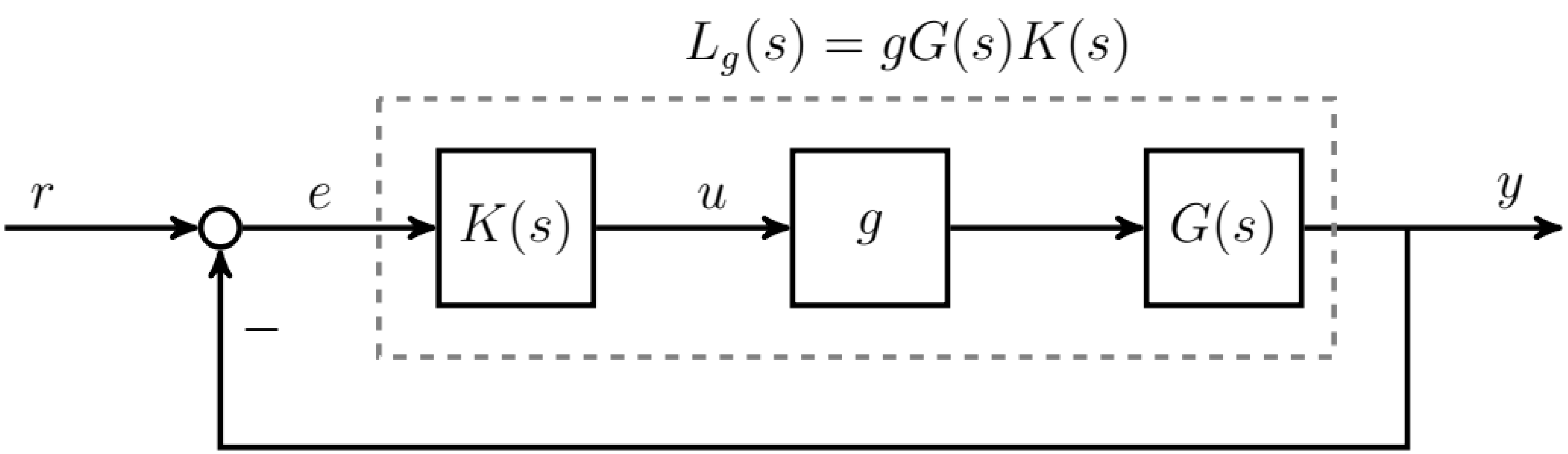
$$L(s) = G(s)K(s) = \frac{2}{s^3+2s^2+3s-1}.$$

Consider the gain margin. For this loop:

- Upper Gain Margin: $\bar{g} = 3.5 \text{ at } \omega_1 = 1.73 \frac{rad}{sec}$
- Lower Gain Margin: $\underline{g} = 0.5 \text{ at } \omega_2 = 2.00 \frac{rad}{sec}$

The upper gain margin causes $1 + \bar{g}L(j\omega_1) = 0 \Leftrightarrow \bar{g}\, L(j\omega_1) = -1$.

(And similar for the lower gain margin.)



Margins on a Nyquist Plot

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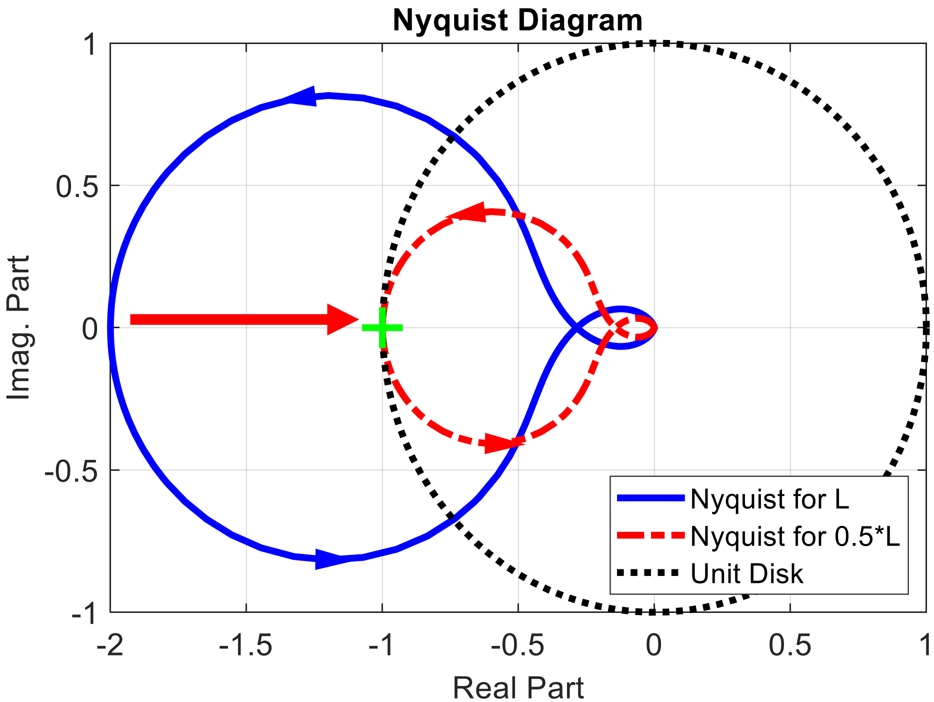

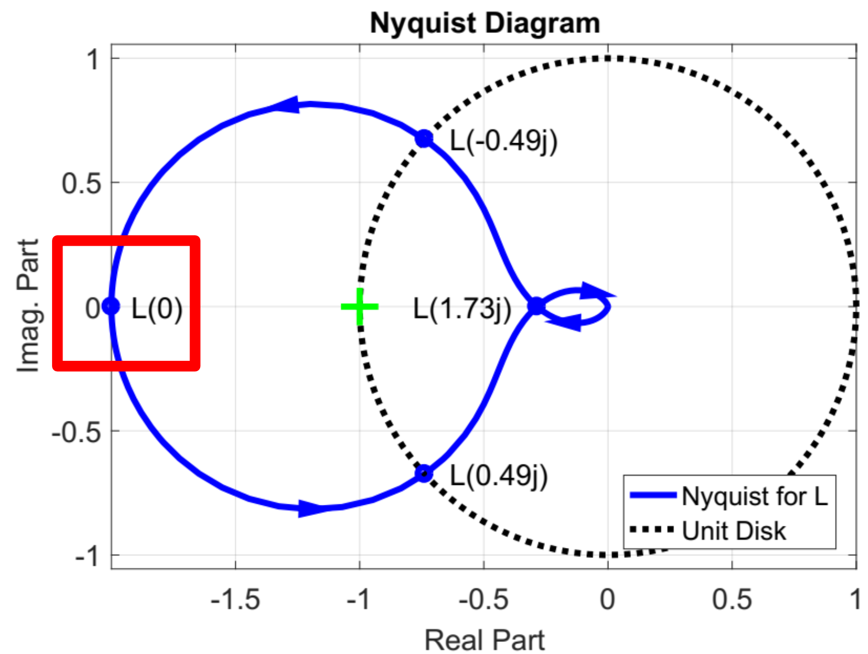

Margins on a Nyquist Plot

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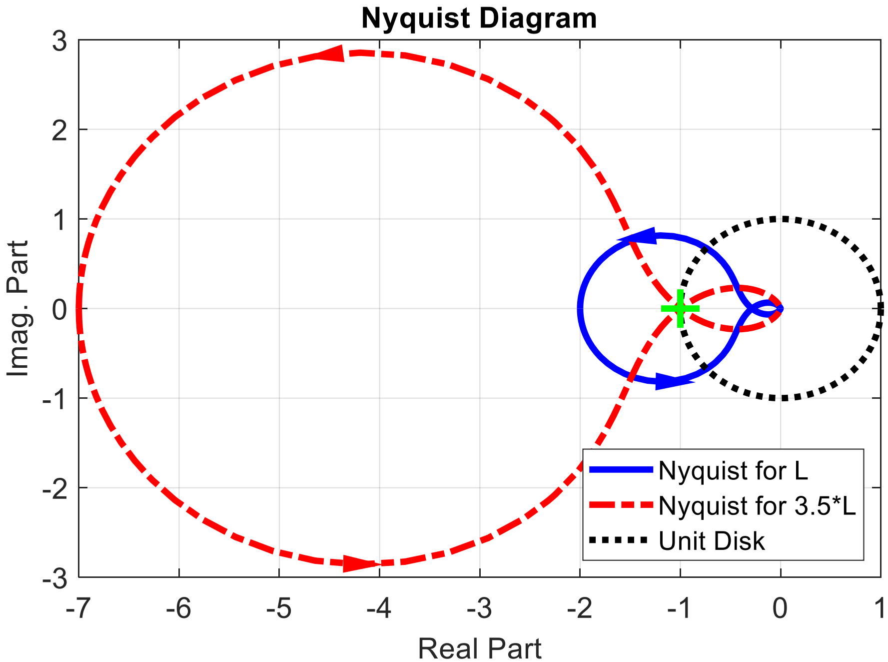
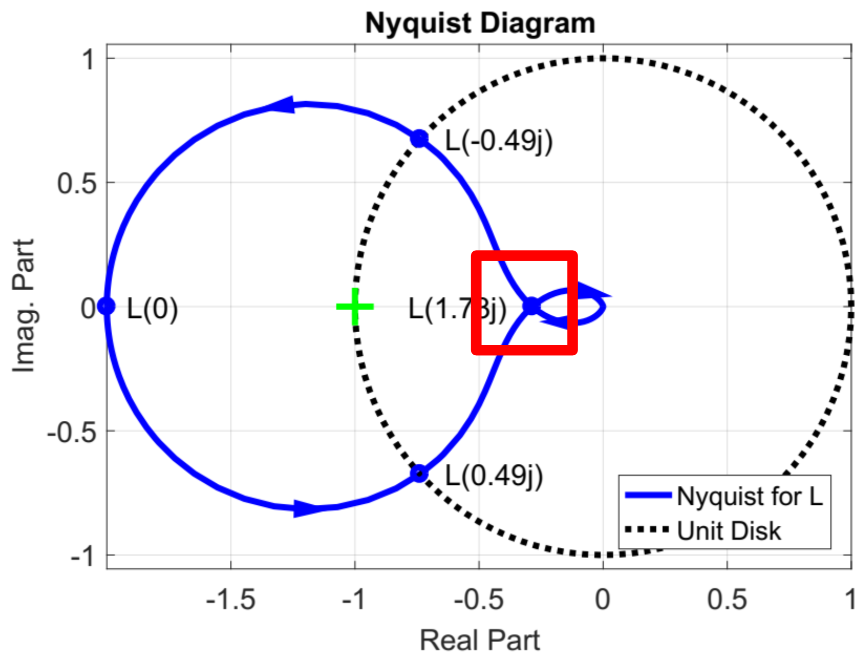




Margins on a Nyquist Plot

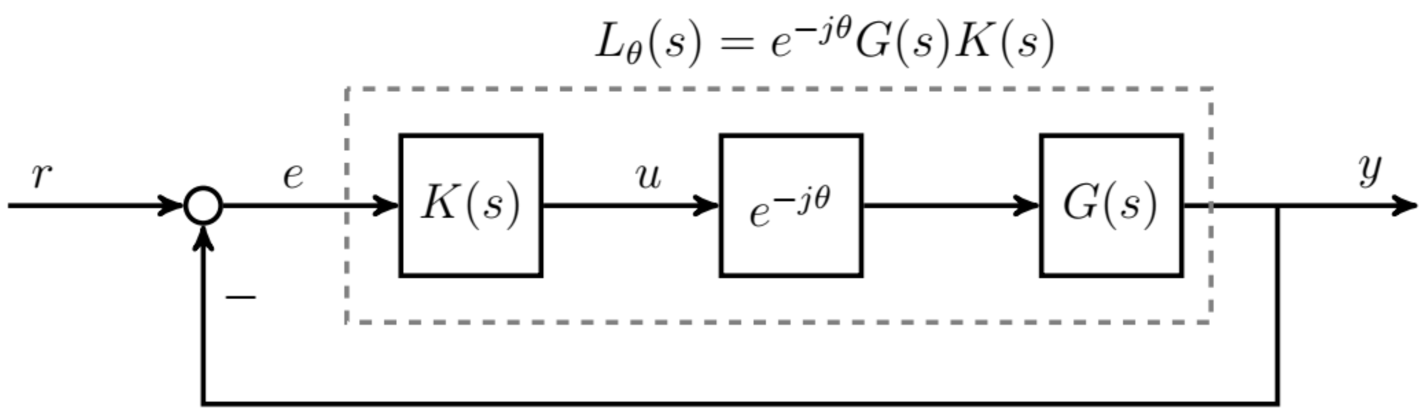
Consider the feedback system with:

$$L(s) = G(s)K(s) = \frac{2}{s^3+2s^2+3s-1}.$$

Consider the phase margin. For this loop:

- Phase Margin: $\bar{\theta} = 42.4^o$ at $\omega_1 = 0.49\frac{rad}{sec}$

The phase margin causes $1 + e^{-j\bar{\theta}}L(j\omega_1) = 0 \Leftrightarrow e^{-j\bar{\theta}}L(j\omega_1) = -1$.



Margins on a Nyquist Plot

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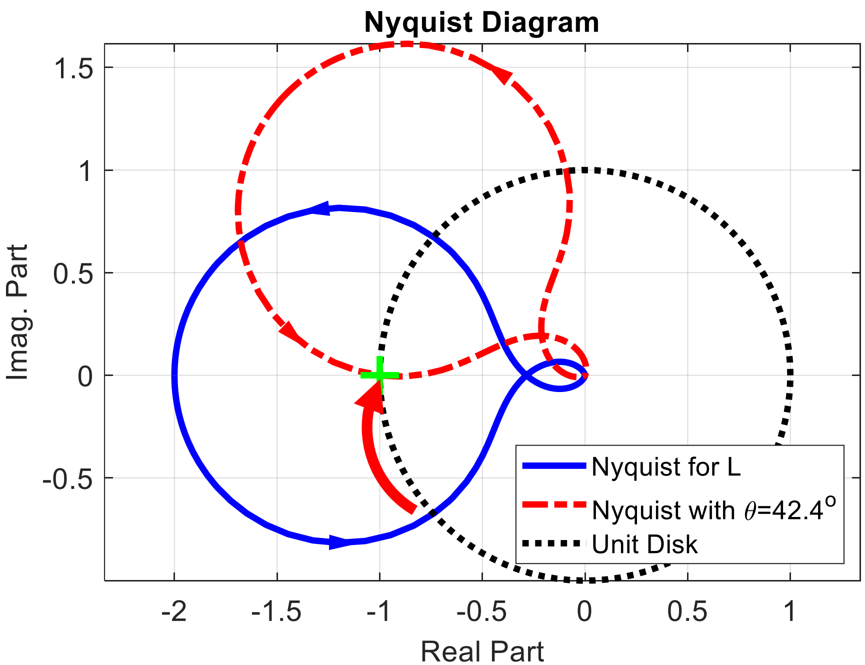
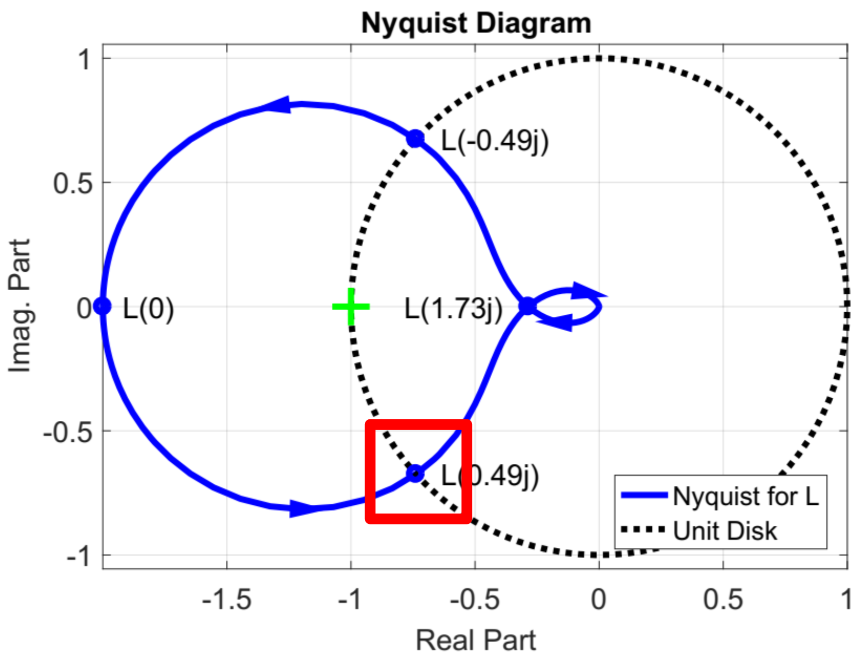
Consider the phase margin. For this loop:

- Phase Margin: $\bar{\theta} = 42.4^o$ at $\omega_1 = 0.49\frac{rad}{sec}$

Nyquist of $e^{-j\theta}L(j\omega)$ with $\theta = \bar{\theta}$.





Margins on a Nyquist Plot

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$$L(s) = G(s)K(s) = \frac{2}{s^3+2s^2+3s-1}.$$

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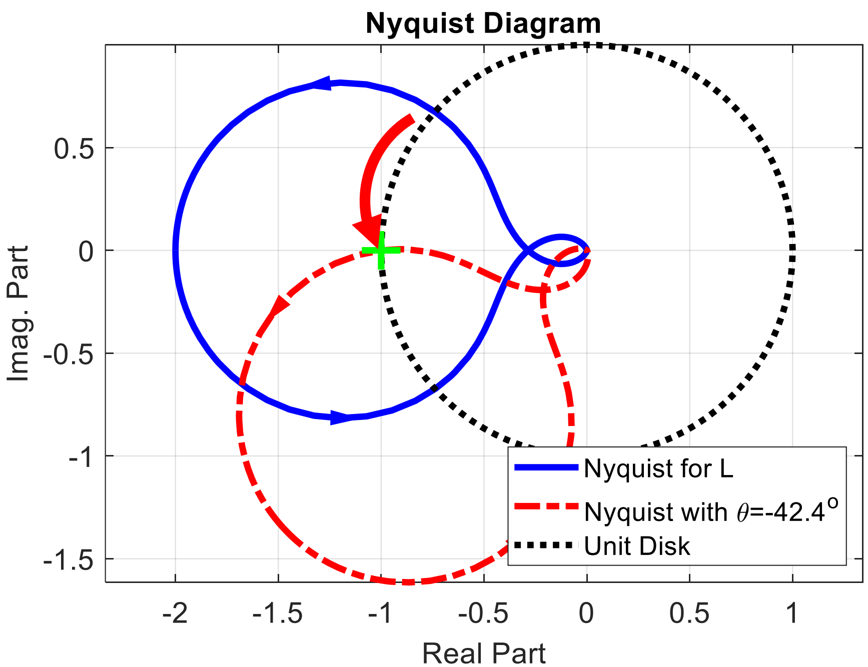
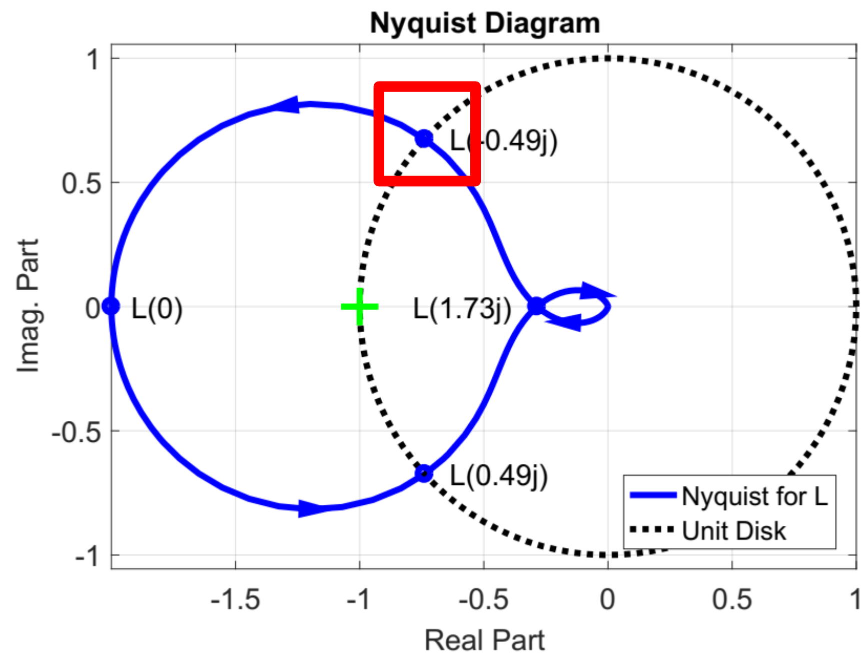
- Phase Margin: $\bar{\theta} = 42.4^o \text{ at } \omega_1 = 0.49\frac{rad}{sec}$

Nyquist of $e^{-j\theta}L(j\omega)$ with $\theta = -\bar{\theta}$.

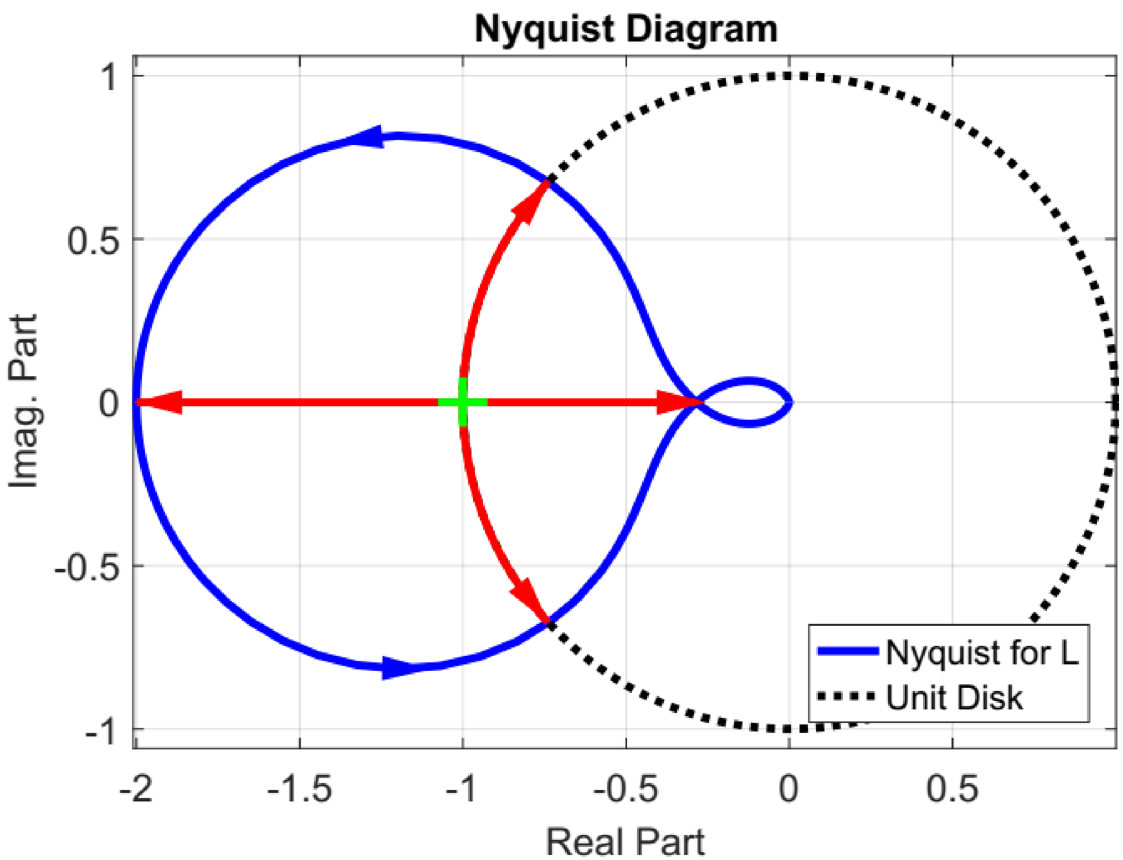




Gain/Phase Margins On Nyquist Plots

Gain and phase variations can cause the Nyquist curve to change from the “correct" number of -1 encirclements (stable) to an “incorrect" number of encirclements (unstable).

Gain and phase margins measure the distance from the Nyquist curve to the -1 point along two specific directions.



Disk Margin

Minimum distance to the critical -1 point:

$$d_{min} := \min_{\omega} |1 + L(j\omega)|$$

As a rule of thumb, the closed-loop should have $d_{min} \geq 0.4$.

This implies gain and phase margins of at least [0.71,1.61] and 23°.

Also implies $S(s) = \frac{1}{1+L(s)}$ satisfies $\max_{\omega} |S(j\omega)| = \frac{1}{d_{min}} \leq 2.5$.