ECE 486: Control Systems

Lecture 10A: Dominant Pole Approximation

Dominant-Pole Approximation

The dominant poles of a higher-order system are the slowest poles (largest time constant).

We can often approximate a higher-order system by a:

1. First-order approximation if the dominant pole is real
2. Second-order approximation if the dominant pole(s) are a complex pair.

The approximation is accurate if the dominant pole(s) are significantly slower than the remaining poles.

(Dominant pole time constant is 5x larger than other poles)

Problem 1

For each system:

- Construct a first-order or second-order approximation from the dominant pole.
- Do you expect the dominant pole approximation to be accurate?

For system A: Roughly sketch the unit step response of the dominant pole approximation. Note the final time, settling time, and overshoot (if underdamped).

$$G_A(s) = \frac{5000}{(s+2)(s+20)(s^2+20s+500)}$$

$$G_B(s) = \frac{24}{(s+1)(s+2)^2(s+3)}$$

Solution 1A

- Construct a first-order or second-order approximation from the dominant pole.
- Do you expect the dominant pole approximation to be accurate?
- Roughly sketch the unit step response of the dominant pole approximation. Note the final time, settling time, and overshoot (if underdamped).

$$G_A(s) = \frac{5000}{(s+2)(s+20)(s^2+\boxed{20}s+500)}$$

$\rho < 1 \rightarrow -\rho\omega_n$ (Real)

$2\rho\omega_n$

$s = \boxed{-2}, -20, \boxed{-10 \pm 20j}$

Dominant Pole

$\tau = 1/2$ sec

$G_A(0) = \frac{5000}{2 \cdot 20 \cdot 500} = \boxed{1/4}$

$\boxed{\hat{G} = \frac{b_0}{s+2}}$

$\hat{G} = \frac{0.5}{s+2}$

$\hat{G}(0) = \frac{1}{4}$

$\frac{1}{4} = \hat{G}(0) = \frac{b_0}{2}$

$b_0 = \frac{2}{4} = \frac{1}{2}$

Accurate because -2 is much slower than $-10 \pm 20j$

Solution 1B

- Construct a first-order or second-order approximation from the dominant pole.
- Do you expect the dominant pole approximation to be accurate?

$$G_B(s) = \frac{24}{(s+1)(s+2)^2(s+3)}$$

$G_B(0) = \frac{24}{2^2 \cdot 3} = 2$

$s = \underbrace{-1}_{\text{slowest, dominant}}, -2, -2, -3$

$\hat{G} = \frac{2}{s+1}$

Poor approximation
$-1, -2, -2, -3$
are all of similar time scale

$G_C(s) = \frac{15}{(s+1)^2(s+10)}$

$s = \underbrace{-1, -1}_{\text{Dominant}}, -10$

$\hat{G} = \frac{b_0}{(s+1)^2}$

$y_p + c_1 e^{-t} + c_2 t e^{-t}$

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Lecture 10B: Integrator Anti-windup

Key Takeaways

This lecture describes impact of actuator saturation and rate limits. These limits:

- Cause slower speed of response and
- Can lead to overshoot and oscillations if the controller does not properly account for the limits.

Anti-windup is one method to reduce the effect of saturation.

- It will prevent overshoot and oscillations.
- However, it does not change the slower speed of response which is a physical limit of the actuators.

Problem 2

Consider the following plant and PI controller:

$$\dot{y}(t) + 4y(t) = 2u(t) \qquad\qquad u(t) = 5\,e(t) + 20\int_0^t e(\tau)\,d\tau$$

A) What is the ODE that models the closed-loop from *r* to *y*?

B) The actuator saturates at $u \in [-3, +3]$. Do you expect saturation to cause any issues if the reference commands are in the range $r \in [-1, +1]$? If yes, then how might you alleviate the issue?

C) Suppose instead that the references are in the range $r \in [-10, +10]$. Can any controller (not just the PI controller above) achieve good reference tracking with this actuator? If not, then how would you re-select the actuator?

Solution 2A

Consider the following plant and PI controller:

$$\dot{y}(t) + 4y(t) = 2u(t) \qquad u(t) = 5\,e(t) + 20\int_0^t e(\tau)\,d\tau$$

A) What is the ODE that models the closed-loop from *r* to *y*?

$$\ddot{y} + 4\dot{y} = 2\dot{u} = 2[5\dot{e} + 20e]$$

$$e = r - y$$

$$\ddot{y} + (4+10)\dot{y} + 40y = 10\dot{r} + 40r$$

$$T_{r\to y}(s) = \frac{10s + 40}{s^2 + 14s + 40}$$

Solution 2B

Consider the following plant and PI controller:

$$\dot{y}(t) + 4y(t) = 2u(t) \qquad u(t) = \boxed{5\,e(t)} + \boxed{20 \boxed{\int_0^t e(\tau)\,d\tau}}$$

B) The actuator saturates at $u \in [-3, +3]$. Do you expect saturation to cause any issues if the reference commands are in the range $r \in [-1, +1]$? If yes, then how might you alleviate the issue?

$u = 5e + 20z$

$\dot{z} = e \leftrightarrow z = \int_0^t e(\tau)\,d\tau$

$$u(0) = 5(\underbrace{r(0)}_{1} - \underbrace{y(0)}_{0}) + 20\underbrace{\int_0^0 e(\tau)\,d\tau}_{=0}$$

$u(0) = 5$

$$\dot{z} = \begin{cases} 0 & \text{if } u \geq 3 \text{ and } e \geq 0 \\ 0 & \text{if } u \leq -3 \text{ and } e \leq 0 \\ e & \text{else} \end{cases}$$

Clamping / Anti-windup

$\leftarrow$ Turn off integration when $|u| \geq 3$ (and error makes $|u|$ grow more)

Solution 2C

Consider the following plant and PI controller:

$$\boxed{\dot{y}(t) + 4y(t) = 2u(t)} \qquad u(t) = 5\,e(t) + 20\int_0^t e(\tau)\,d\tau$$

C) Suppose instead that the references are in the range $r \in [-10, +10]$. Can any controller (not just the PI controller above) achieve good reference tracking with this actuator? If not, then how would you re-select the actuator?

$u \in [-3, 3]$

In steady-state

$\bar{y} = G(0)\,\bar{u}$, $G(0) = 1/2$

$G(s) = \frac{2}{s+4}$

$G(0) = \frac{2}{4} = \frac{1}{2}$

$\bar{y} = G(0)\,\bar{u} = \frac{1}{2} \cdot 3 = 1.5$

$4\bar{y} = 2\bar{u} = 2 \cdot 3 = 6$

$\bar{y} = 6/4 = 1.5$

Embedded Control

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Lecture 10C: Control Law Implementation

Key Takeaways

It is common to implement controllers on a microprocessor.

This lecture discusses some of the details associated with this implementation:

- Sample a measurement at specific (discrete) time invervals
- Update the control input *u* at each sample time.
- Hold the control input *u* constant until the next update.

The update equation is chosen to approximate the properties of the designed (ODE) controller. The update equation can be implemented on a microprocessor with a few lines of code.

Problem 3

Consider the following plant and PI controller:

$$2\dot{y}(t) + 6y(t) = 8u(t) \qquad\qquad u(t) = 2.5\, e(t) + 9 \int_0^t e(\tau)\, d\tau$$

A) What sampling time Δt would you recommend for a discrete-time implementation?

B) The value of u(t) at *t=Δt* is:

$$u(\Delta t) = 2.5\, e(\Delta t) + 9 \int_0^{\Delta t} e(\tau)\, d\tau$$

Approximate $u_1 := u(\Delta t)$ in terms $e_0 := e(0)$ and $e_1 := e(\Delta t)$.

C) The computations in the discrete-time update are not instantaneous and require some time. How can this be modeled?

Solution 3A

$$2\ddot{y}(t) + 6\dot{y}(t) = 8\dot{u}(t) \qquad u(t) = 2.5\, e(t) + 9 \int_0^t e(\tau)\, d\tau$$

A) What sampling time Δt would you recommend for a discrete-time implementation?

$$2\ddot{y} + 6\dot{y} = 8\dot{u} = 8\,\overbrace{[2.5(\dot{r}-\dot{y}) + 9(r-y)]}$$

$$2\ddot{y} + \underbrace{[6+20]}_{26}\dot{y} + 72y = 20\dot{r} + 72r$$

$$T_{r\to y}(s) = \frac{20s+72}{2s^2+26s+72}$$

$$s_{1,2} = -4, -9 \text{ rad/sec}$$

$$\tau_{1,2} = 1/4,\ \boxed{1/9} \text{ sec}$$

$$\Delta t = \frac{1/4}{10} = 1/40 \text{ sec} \quad X$$

$$\boxed{\Delta t = \frac{1/9}{10} \approx 1/90 \text{ sec} \approx 1/100 \text{ sec} = 10 \text{ msec}}$$

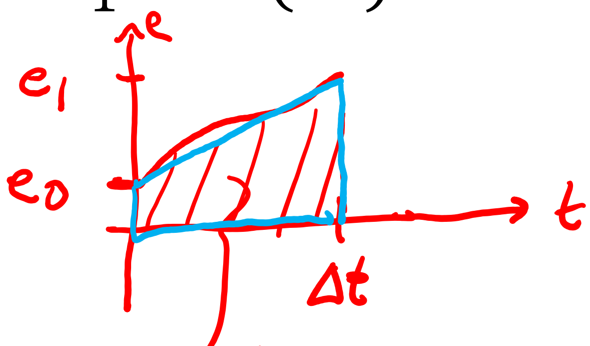
Solution 3B

$$2\dot{y}(t) + 6y(t) = 8u(t) \qquad \boxed{u(t) = 2.5\, e(t) + 9 \int_0^t e(\tau)\, d\tau}$$

B) The value of u(t) at *t=Δt* is:

$$u(\Delta t) = 2.5\, e(\Delta t) + 9 \underline{\int_0^{\Delta t} e(\tau)\, d\tau}$$

Approximate $u_1 := u(\Delta t)$ in terms $e_0 := e(0)$ and $e_1 := e(\Delta t)$.



$$\int_0^{\Delta t} e(\tau)\, d\tau \simeq \frac{1}{2}(e_0 + e_1) \cdot \Delta t$$

memory

$$u_1 = 2.5\, e_1 + 9 \left[\frac{1}{2}(e_0 + e_1) \cdot \Delta t \right]$$

$$u_2 = 2.5\, e_2 + 9 \left[\frac{1}{2}(e_1 + e_2) \cdot \Delta t \right]$$

e   e

memory

Solution 3C

$$2\dot{y}(t) + 6y(t) = 8u(t) \qquad\qquad u(t) = 2.5\, e(t) + 9 \int_0^t e(\tau)\, d\tau$$

C) The computations in the discrete-time update are not instantaneous and require some time. How can this be modeled?