Reading Assignment:
**FPE**, Sections 6.3, 6.7.1–6.7.6.

Problems:

1. Consider the transfer function
   \[ H(s) = \frac{1}{s + a}, \]
   where \( a > 0 \). Prove that the Nyquist plot of \( H \) is a circle of radius \( \frac{1}{2a} \) centered at the point \( \left( \frac{1}{2a}, 0 \right) \).

2. For the two plant transfer functions given below, use the Nyquist stability criterion to determine all values of the feedback gain \( K \) that stabilize the closed-loop system.
   
   (a) \[ G(s) = \frac{1}{(s + 2)(s + 5)} \]
   (b) \[ G(s) = \frac{1}{(s + 2)(s^2 + 2s + 5)} \]

   *Instructions:* To draw the Nyquist plot, use the Bode plots of \( G(s) \). Explain all steps in arriving at the Nyquist plot. Start with hand-sketched Bode plots. You can then generate more accurate Bode plots with MATLAB to get exact numerical values if necessary. *The use of MATLAB for drawing the Nyquist plot is not allowed* (except to check your work at the end). It is also recommended that you check your results with Routh-Hurwitz stability criterion.

3. For the two transfer functions and gain values given below, use the Nyquist plot to find the gain and the phase margins:
   
   (a) \[ G(s) = \frac{1}{(s - 1)(s + 2)(s + 4)}, \quad K = 10; \]
   (b) \[ G(s) = \frac{1}{(s + 1)^3}, \quad K = 3. \]

   *Instructions:* Follow the same instructions as in the previous problem. *You may (and should) use MATLAB for drawing a more accurate Nyquist plot to find the GM and the PM, but you must explain all your work on the hand-sketched Nyquist plot.*