ECE 486 **Assignment # 4**

Issued: September 18 Due: September 27, 2018

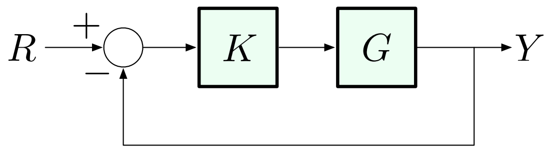
Reading Assignment:

FPE, Sections 4.1–4.3, 5.1.

Problems:

1. Consider the following feedback system, where K is a constant gain and

$$G(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}:$$



Using the Routh–Hurwitz criterion, show that the system is stable for $-1 < K < 3$ and unstable for $K \geq 3$. (This illustrates the destabilizing effect of feedback when the gain is too high.)

2. Consider the same feedback configuration as in Problem 1, but now with $K(s)$ and $G(s)$ unknown transfer functions. Suppose that we know that the transfer function from R to Y is

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

for some parameters $\zeta > 0$ and $\omega_n > 0$.

(i) Based on this information, find the forward gain $K(s)G(s)$.

(ii) Determine the system type and discuss what it implies about the closed-loop system's steady-state tracking capability.

3. The *speed* y of a DC motor satisfies $\dot{y} + 60y = 600v - 1500d$, where v is the armature voltage — the input to the system — and d is a load.

Suppose v is defined via the PI control law,

$$v = K_{\mathrm{P}} e + K_{\mathrm{I}} \int_0^t e(s)\,\mathrm{d}s.$$

where $e = r - y$, as usual, with r the reference speed.

(i) Define a disturbance process $\tilde{d}$ so that the model can be expressed in the 'input disturbance' form,

$$Y = G_p(V + \tilde{D})$$

where G_p is the plant transfer function.

(ii) Draw a complete block diagram showing the plant, compensator (controller), input, output, and the location of disturbance and reference inputs.

(iii) Compute the transfer function $H_1(s) = Y(s)/R(s)$ for arbitrary choices of $K_{\mathrm{P}}, K_{\mathrm{I}}$, and note the value of $H_1(0)$.

(iv) Repeat with the transfer function $H_2(s) = Y(s)/D(s)$ (not $\widetilde{D}(s)$).

(v) Compute values of $K_{\mathrm{P}}, K_{\mathrm{I}}$ so that the following closed loop specifications hold for a step reference input: no more than 5% overshoot, $t_s^{5\%} = 1$ sec. and no undershoot (no zeros in RHP).

Note: In this part of the problem you take $d \equiv 0$.

4. Set up the listed characteristic equations in the form suited to Evans' root-locus method, as described in Section 5.1. In each case, express $a(s)$, $b(s)$, and K in terms of the original parameters. Remember that the polynomials $a(s)$ and $b(s)$ must be monic, and the degree of $b(s)$ no greater than the degree of $a(s)$.

(i) $s + 1/\tau^2 = 0$ vs. parameter τ.

(ii) $(s + R)^3 + s/T = 0$ vs. parameter T.

(iii) $(s + R)^3 + s/T = 0$ vs. parameter R, if possible. If not possible, explain why.