Reading: FPE, Sections 7.5–7.8, notes on state-space control by Prof. Belabbas (documents section of the class website). Note: some material is developed differently in the book than in class. In these cases, the knowledge of the class approach is mandatory while the knowledge of the book approach is optional. You will see how to solve Prob. 5 on Tuesday, or you can read the class notes.

Problems: (you can use MATLAB to perform necessary matrix computations)

1. Consider the system

\[ \dot{x} = \begin{pmatrix} 0 & -1 & 2/3 \\ -1 & -2 & 1 \\ 0 & -3 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} u \]

a) Using the procedure described in class (based on converting to CCF), design a full-state feedback law \( u = -Kx \) which places the closed-loop poles at \(-10\) and \(-10 \pm 5j\).

b) If the real parts of the desired closed-loop poles were \(-100\) instead of \(-10\), what would happen to the control gains? Give a conceptual answer to this question, without making any extra calculations.

2. Consider the system

\[ \dot{x} = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix} x + \begin{pmatrix} 1 \\ b \end{pmatrix} u, \quad y = \begin{pmatrix} 1 & 1 \end{pmatrix} x \]

a) Derive the transfer function using the formula given in class, keeping \( b \) a general constant.

b) Show that for \( b = 0 \), there is a pole/zero cancellation in the transfer function and loss of controllability in the system.

3. Determine (from the observability matrix) whether or not the following systems are observable.

a) \( \dot{x} = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix} x, \quad y = x_2 \)

b) \( \dot{x} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & -5 \\ 3 & 3 & -2 \end{pmatrix} x, \quad y = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} x \)

4. For the system

\[ \dot{x} = \begin{pmatrix} 0 & -1 & 2/3 \\ -1 & -2 & 1 \\ 0 & -3 & 1 \end{pmatrix} x, \quad y = x_2 \]

design an observer with observer poles (poles of \( A-LC \)) placed at \(-20\) and \(-20 \pm 2j\). (Follow the procedure described in class, which involves solving the corresponding pole placement problem for an auxiliary system \( \dot{x} = Fx + Gu \).)

5. Consider the system

\[ \dot{x} = \begin{pmatrix} 0 & -1 & 2/3 \\ -1 & -2 & 1 \\ 0 & -3 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} u, \quad y = x_2 \]

Combine the results of the Problems 1 and 4 to obtain a controller in the form of dynamic output feedback (observer plus estimated state feedback). Write down the state-space model of the controller as well as its transfer function (you can use MATLAB command \texttt{ss2tf} to compute the latter).