**Problems:**

1. Consider the transfer function

\[ G(s) = \frac{s + 1}{s^2 + s + 1} \]

Use MATLAB to compare the \( M_p \) from the step response of the system for \( \alpha = 0.01, 0.1, 1, 10, \) and 100 with the \( M_r \) from the frequency response for the same values of \( \alpha = \). Is there a correlation between \( M_p \) and \( M_r \)?

**Solution:**

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>Resonant peak ( M_r )</th>
<th>Overshoot ( M_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>98.8</td>
<td>54.1</td>
</tr>
<tr>
<td>0.1</td>
<td>9.93</td>
<td>4.94</td>
</tr>
<tr>
<td>1</td>
<td>1.46</td>
<td>0.30</td>
</tr>
<tr>
<td>10</td>
<td>1.16</td>
<td>0.16</td>
</tr>
<tr>
<td>100</td>
<td>1.15</td>
<td>0.16</td>
</tr>
</tbody>
</table>

As \( \alpha \) is reduced, the resonant peak in frequency response increases. This leads us to expect extra peak overshoot in transient response. This effect is significant in case of \( \alpha = 0.01, 0.1, 1 \), while the resonant peak in frequency response is hardly changed in case of \( \alpha = 10 \). Thus, we do not have considerable change in peak overshoot in transient response for \( \alpha \geq 10 \). The response peak in frequency response and the peak overshoot in transient response are correlated.
2. Consider the transfer function

\[
G(s) = \frac{1}{\left(\frac{s}{p} + 1\right)(s^2 + s + 1)}
\]

Draw the Bode plots for \( p = 0.01, 0.1, 1, 10, \) and 100. What conclusions can you draw about the effect of the pole at \(-p\) on the bandwidth compared with the bandwidth for the second-order system without this pole? MATLAB use is allowed.

**Solution:**

<table>
<thead>
<tr>
<th>( p )</th>
<th>Additional pole ((-p))</th>
<th>Bandwidth ( (\omega_{BW}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>-0.01</td>
<td>0.013</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.1</td>
<td>0.11</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>1.0</td>
</tr>
<tr>
<td>10</td>
<td>-10</td>
<td>1.5</td>
</tr>
<tr>
<td>100</td>
<td>-100</td>
<td>1.7</td>
</tr>
</tbody>
</table>

As \( p \) is reduced, the bandwidth decreases. This leads us to expect slower time response and additional rise time. This effect is significant in case of \( p = 0.01, 0.1, 1 \), while the bandwidth is hardly changed in case of \( p = 10 \). Thus, we do not have considerable change in rise time for \( p \geq 10 \). Bandwidth is a measure of the speed of response of a system, such as rise time.
3. Consider the following problem: for the system $G(s) = \frac{1}{s^2}$, design a lead controller that gives PM $\approx 90^\circ$ and $\omega_{BW} \approx 0.5$. This homework problem asks you to check and improve the design given in class.

a) For the controller:

$$KD(s) = \frac{1}{16} \frac{s^2 + 1}{\frac{s}{2} + 1}$$

compute the PM, open-loop crossover frequency $\omega_c$, and closed-loop bandwidth $\omega_{BW}$. Plot the closed-loop step response. Explain the reasons why this design didn’t fully meet the specs.

b) Improve the design to obtain PM and $\omega_{BW}$ closer to the specs. Does the new closed-loop step response show better damping?

**Solution:**

$$G(s) = \frac{1}{s^2}$$

a)

$$KD(s) = \frac{1}{16} \frac{s^2 + 1}{\frac{s}{2} + 1}$$
Using the bode plot attached, we can see that PM = 63.8° and ω_c = 0.606. We can see that PM is far from 90°. For this case, the whole PM should be provided by controller. It means that

\[ \sin \phi_m = \frac{p - z}{p + z} \]

where \( \phi_m \) is the maximum phase provided by Lead controller and \( p \) and \( z \) are lead pole and lead zero, respectively. We need either \( z \approx 0 \) or \( \frac{p}{z} \to \infty \).

b) To improve the above design, we need to enlarge \( \frac{p}{z} \), an example would be:

\[ KD(s) = \frac{2.5s + 0.095}{s + 3.8} \]

which improves the PM to 72°.
An “extreme” design is also provided by

\[ KD(s) = 5 \frac{s}{s + 10}. \]

The bode and time response is attached.
4. For the two plant transfer functions given below, use the Nyquist stability criterion to determine all values of the feedback gain $K$ that stabilize the closed-loop system.

a) $G(s) = \frac{1}{(s + 2)(s + 4)}$  

b) $G(s) = \frac{1}{(s + 1)(s^2 + s + 1)}$

Instructions: To draw the Nyquist plot, use the Bode plots of $G(s)$. Explain all steps in arriving at the Nyquist plot. Start with hand-sketched Bode plots. You can then generate more accurate Bode plots with MATLAB to get exact numerical values if necessary. The use of MATLAB for drawing the Nyquist plot is not allowed (except to check your work at the end). It is also recommended that you check your results with Routh stability criterion.

Solution:

(a) $G(s) = \frac{1}{(s + 2)(s + 4)}$
\( \omega \to 0 \implies |G(j\omega)| = \frac{1}{8}, \angle G(j\omega) = 0^\circ \)

\( \omega = \sqrt{8} \implies |G(j\omega)| = \frac{1}{6\sqrt{8}}, \angle G(j\omega) = -90^\circ \)

\( \omega \to \infty \implies |G(j\omega)| \to 0, \angle G(j\omega) = -180^\circ \)

\( P : \# \text{ RHP poles} = 0 \)
\( Z : \# \text{ of closed loop RHP poles} \)
\( N : \# \text{ of encirclements of } -1/K \)

\[ G(s) = \frac{1}{(s + 1)(s^2 + s + 1)} \]
\[ \omega \to 0 \Rightarrow |G(j\omega)| = 1, \angle G(j\omega) = 0^\circ \]

\[ \omega = \frac{1}{\sqrt{2}} \Rightarrow |G(j\omega)| = 0.95, \angle G(j\omega) = -90^\circ \]

\[ \omega = \sqrt{2} \Rightarrow |G(j\omega)| = \frac{1}{3}, \angle G(j\omega) = -180^\circ \]

\[ \omega \to \infty \Rightarrow |G(j\omega)| \to 0, \angle G(j\omega) = -270^\circ \]

\( P = 0 \). We can see that if \(-\frac{1}{3} \in (-\frac{1}{3}, 1)\) then \( N > 0 \) and closed-loop system would be unstable for stability we need \(-1 < K < 3\) so that \( N = 0 \)