ECE 486, Fall 2017 PROBLEM SET 7 - SOLUTION Due Thuesday, November 7

Reading: FPE, Sections 6.4-6.6, 6.7.1-6.7.6.

Problems:

1. Consider the transfer function

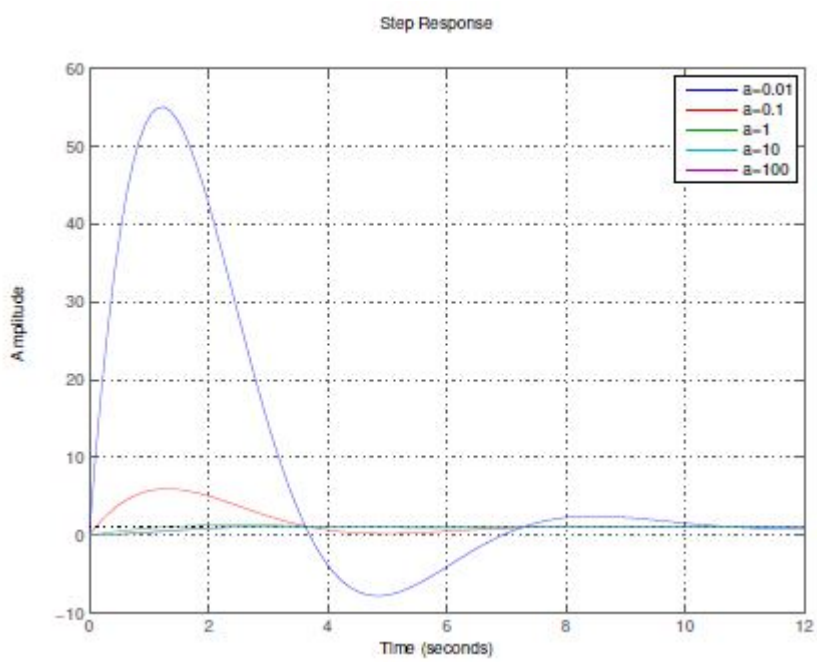
$$G(s) = \frac{\frac{s}{a} + 1}{s^2 + s + 1}$$

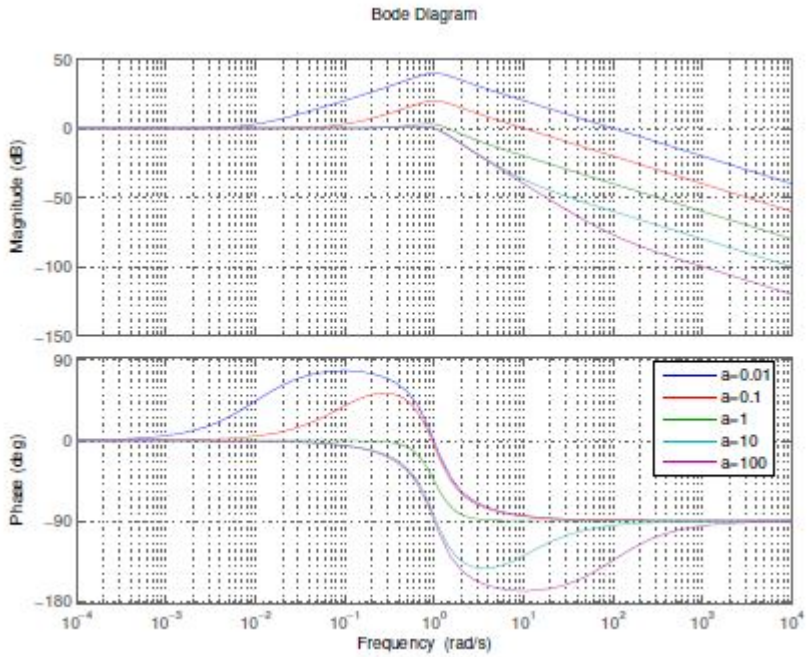
Use MATLAB to compare the M_p from the step response of the system for a =0.01, 0.1, 1, 10, and 100 with the M_r from the frequency response for the same values of a =. Is there a correlation between M_p and M_r?

Solution:

α	Resonant peak (M_r)	Overshoot (M_p)
0.01	98.8	54.1
0.1	9.93	4.94
1	1.46	0.30
10	1.16	0.16
100	1.15	0.16

As α is reduced, the resonant peak in frequency response increases. This leads us to expect extra peak overshoot in transient response. This effect is significant in case of $\alpha = 0.01, 0.1, 1$, while the resonant peak in frequency response is hardly changed in case of $\alpha = 10$. Thus, we do not have considerable change in peak overshoot in transient response for $\alpha \geq 10$. The response peak in frequency response and the peak overshoot in transient response are correlated.




Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
a=0.01
a=0.1
a=1
a=10
a=100

2. Consider the transfer function

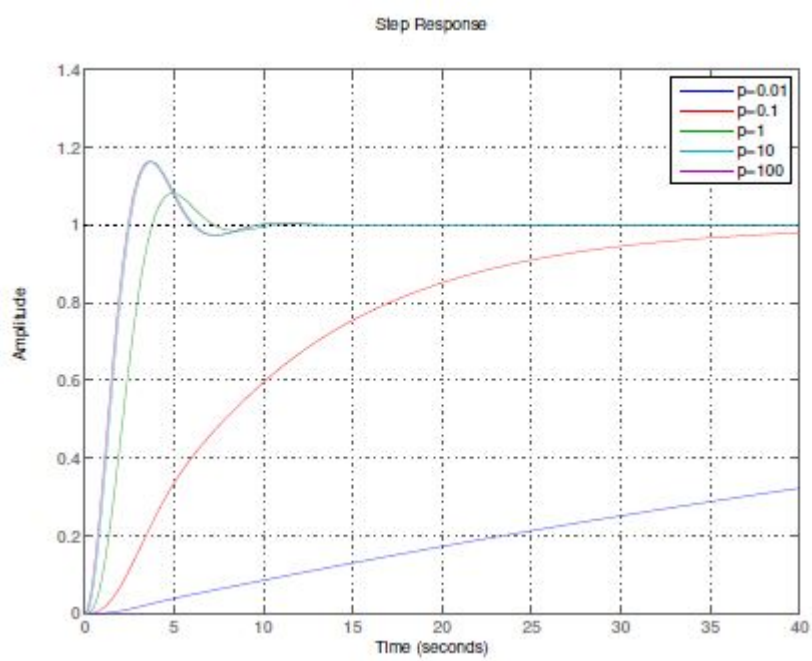
$$G(s) = \frac{1}{\left(\frac{s}{p}+1\right)(s^2+s+1)}$$

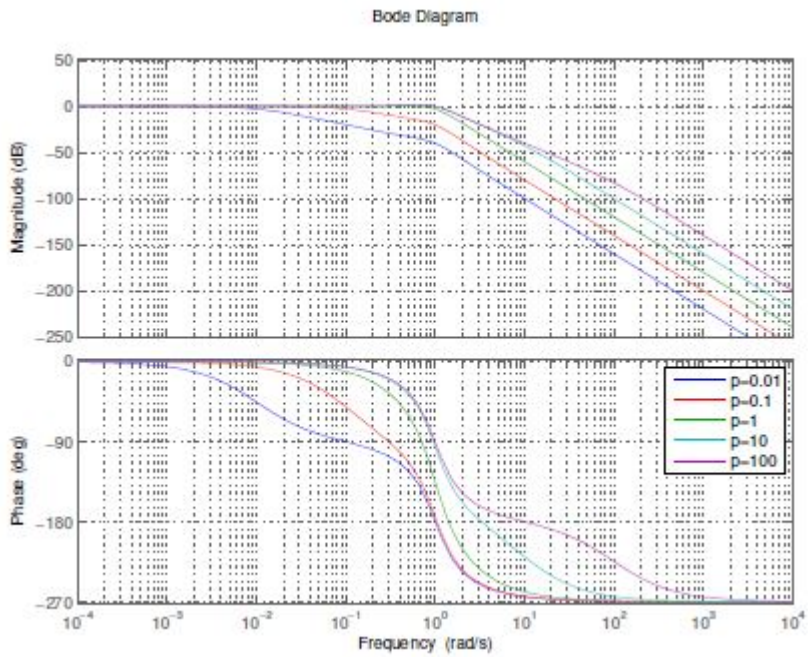
Draw the Bode plots for $p = 0.01$, 0.1, 1, 10, and 100. What conclusions can you draw about the effect of the pole at $-p$ on the bandwidth compared with the bandwidth for the second-order system without this pole? MATLAB use is allowed.

Solution:

p	Additional pole ($-p$)	Bandwidth (ω_{BW})
0.01	-0.01	0.013
0.1	-0.1	0.11
1	-1	1.0
10	-10	1.5
100	-100	1.7

As p is reduced, the bandwidth decreases. This leads us to expect slower time response and additional rise time. This effect is significant in case of $p = 0.01, 0.1, 1$, while the bandwidth is hardly changed in case of $p = 10$. Thus, we do not have considerable change in rise time for $p \geq 10$. Bandwidth is a measure of the speed of response of a system, such as rise time.





3. Consider the following problem: for the system $G(s) = \dfrac{1}{s^2}$, design a lead controller that gives PM $\approx 90°$ and $\omega_{\text{BW}} \approx 0.5$. This homework problem asks you to check and improve the design given in class.

a) For the controller:

$$KD(s) = \frac{1}{16}\frac{\frac{s}{0.1}+1}{\frac{s}{2}+1}$$

compute the PM, open-loop crossover frequency ω_c, and closed-loop bandwidth ω_{BW}. Plot the closed-loop step response. Explain the reasons why this design didn't fully meet the specs.

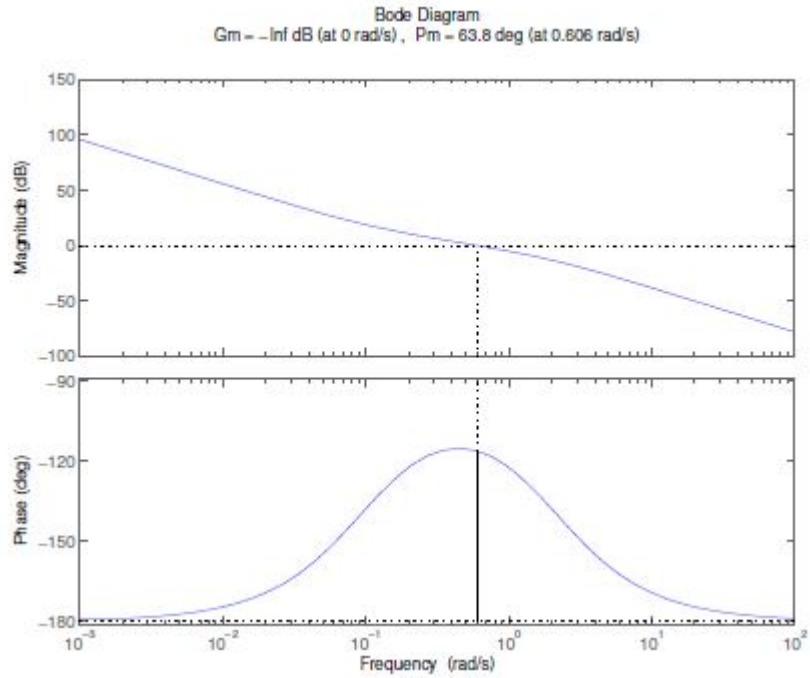
b) Improve the design to obtain PM and ω_{BW} closer to the specs. Does the new closed-loop step response show better damping?

Solution:

$$G(s) = \frac{1}{s^2}$$

a)

$$KD(s) = \frac{1}{16}\frac{\frac{s}{0.1}+1}{\frac{s}{2}+1}$$



Using the bode plot attached, we can see that PM = 63.8° and $\omega_c = 0.606$. We can see that PM is far from 90°. For this case, the whole PM should be provided by controller. It means that

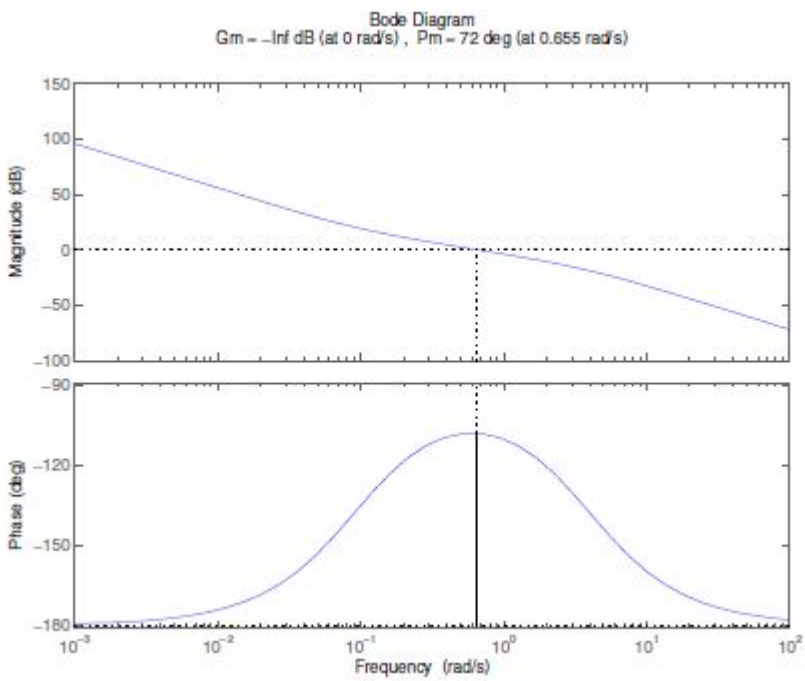
$$\sin \phi_m = \frac{p - z}{p + z}$$

where ϕ_m is the maximum phase provided by Lead controller and p and z are lead pole and lead zero, respectively. We need either $z \approx 0$ or $\frac{p}{z} \to \infty$.

b) To improve the above design, we need to enlarge $\frac{p}{z}$, an example would be:

$$KD(s) = 2.5\frac{s + 0.095}{s + 3.8}$$

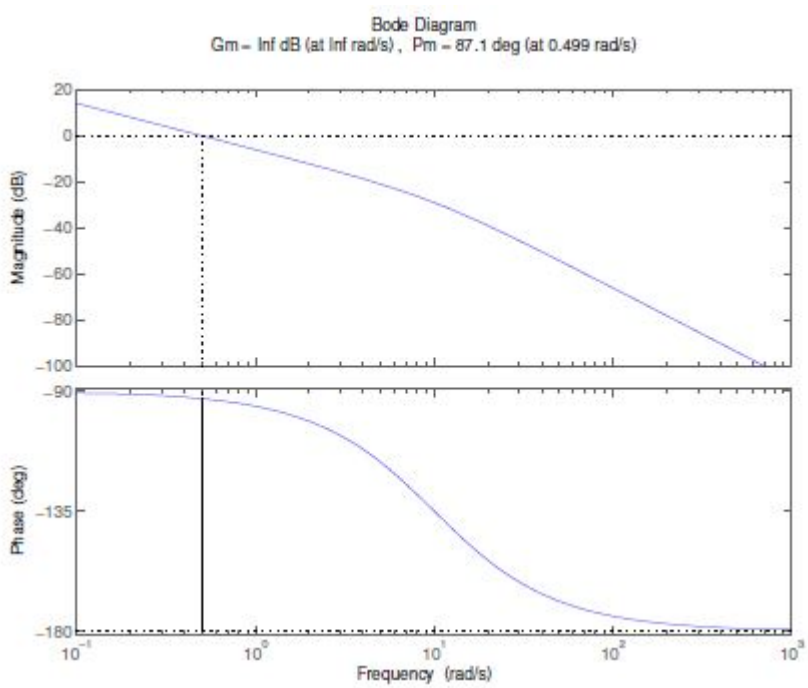
which improves the PM to 72°.

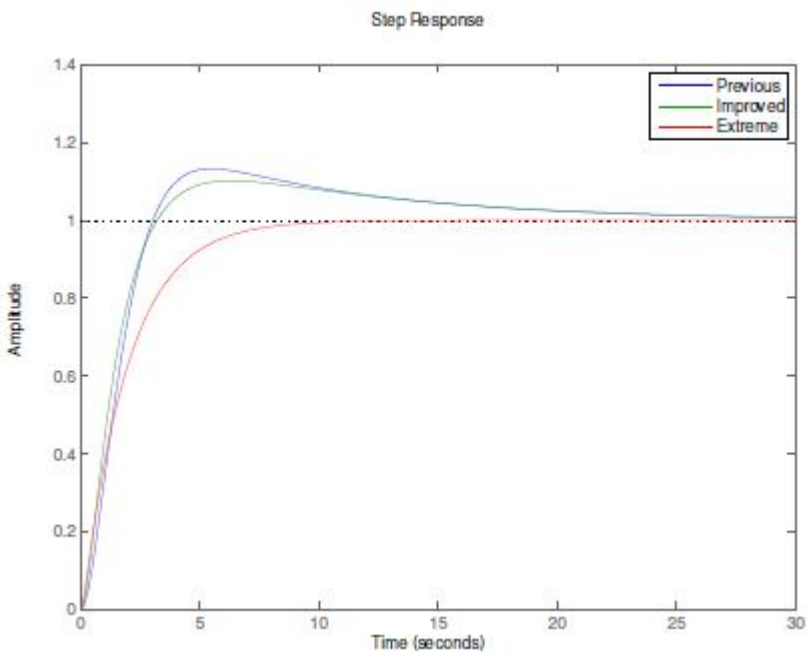


An "extreme" design is also provided by

$$KD(s) = 5\frac{s}{s+10}.$$

The bode and time response is attached.





4. For the two plant transfer functions given below, use the Nyquist stability criterion to determine all values of the feedback gain K that stabilize the closed-loop system.

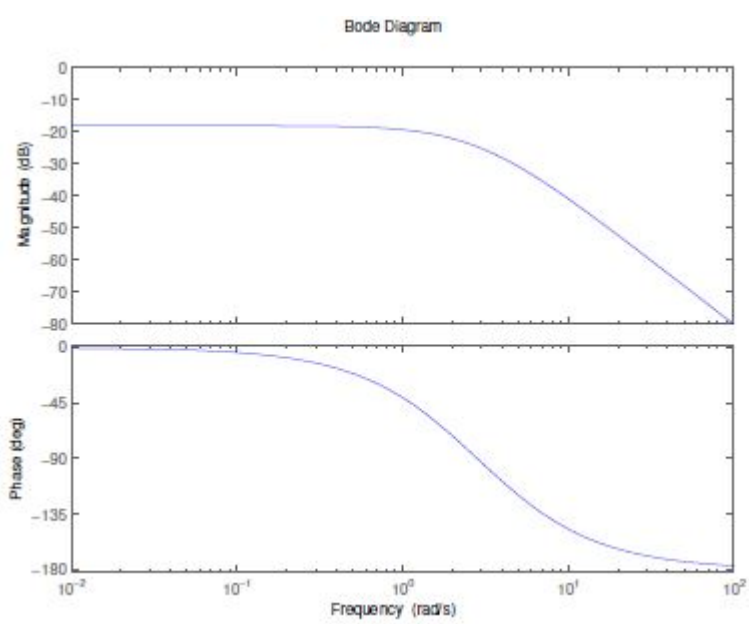
a) $G(s) = \dfrac{1}{(s+2)(s+4)}$ b) $G(s) = \dfrac{1}{(s+1)(s^2+s+1)}$

Instructions: To draw the Nyquist plot, use the Bode plots of $G(s)$. Explain all steps in arriving at the Nyquist plot. Start with hand-sketched Bode plots. You can then generate more accurate Bode plots with MATLAB to get exact numerical values if necessary. *The use of MATLAB for drawing the Nyquist plot is not allowed* (except to check your work at the end). It is also recommended that you check your results with Routh stability criterion.

Solution:

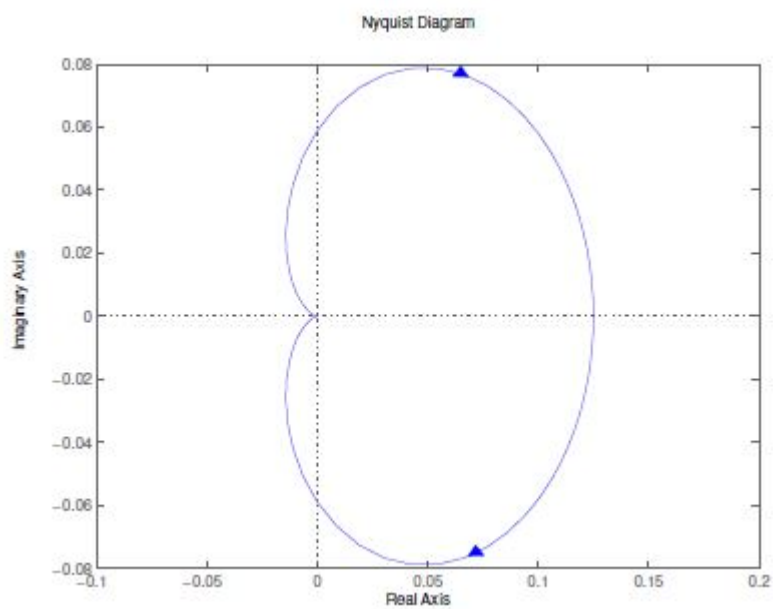
(a)

$$G(s) = \frac{1}{(s+2)(s+4)}$$



$$\omega \to 0 \implies |G(j\omega)| = \frac{1}{8},\ \angle G(j\omega) = 0^\circ$$
$$\omega = \sqrt{8} \implies |G(j\omega)| = \frac{1}{6\sqrt{8}},\ \angle G(j\omega) = -90^\circ$$
$$\omega \to \infty \implies |G(j\omega)| \to 0,\ \angle G(j\omega) = -180^\circ$$

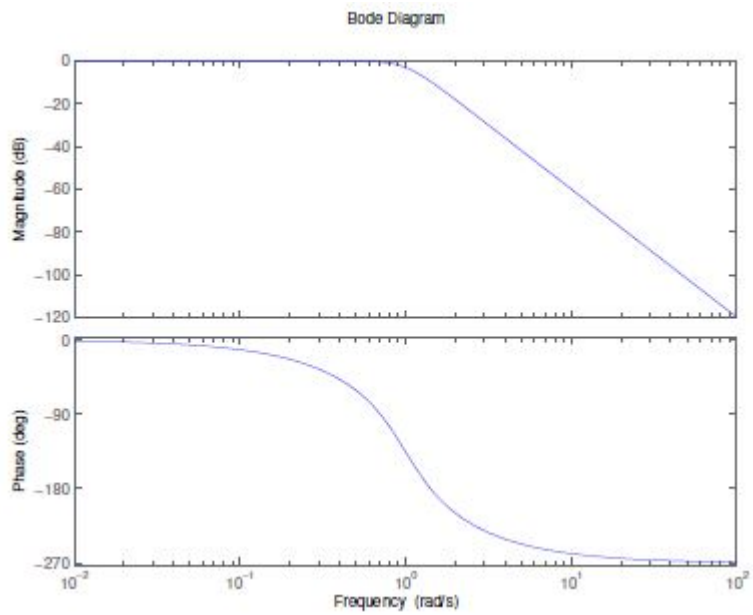
$$P : \#\ \text{RHP poles} = 0$$
$$Z : \#\ \text{of closed loop RHP poles}$$
$$N : \#\ \text{of encirclements of } -1/K$$



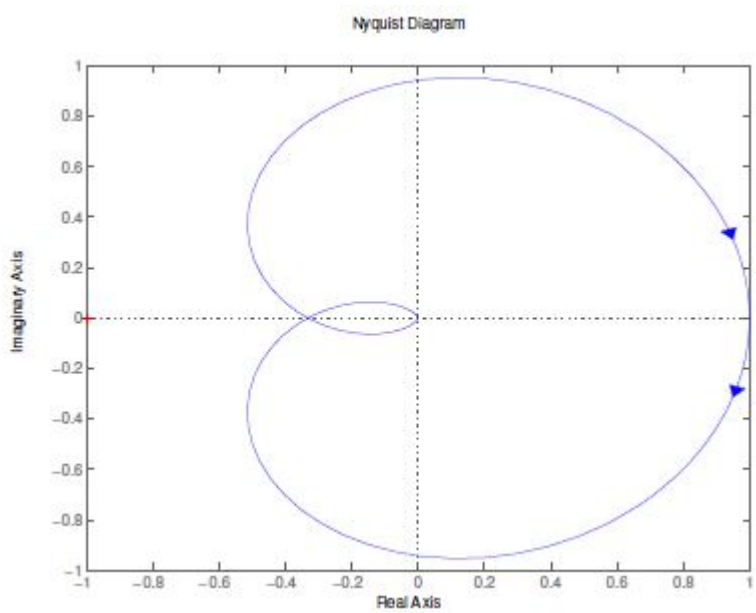
$$Z = N + P \implies Z = N$$
$$\text{if K} > 0 \implies -\frac{1}{K} < 0,\ N = 0 \quad \text{according to Nyquist plot}$$
$$\text{if } -8 < \text{K} \le 0 \implies -\frac{1}{K} > \frac{1}{8},\ N = 0$$
$$\text{if K} < -8 \implies 0 < -\frac{1}{K} < \frac{1}{8} \implies N = 1 \implies Z = 1 \implies \text{unstable closed-loop}$$
$$\implies \text{for stability we need } K \ge -8$$

(b)

$$G(s) = \frac{1}{(s+1)(s^2+s+1)}$$



$$\omega \to 0 \implies |G(j\omega)| = 1,\ \angle G(j\omega) = 0^\circ$$
$$\omega = \frac{1}{\sqrt{2}} \implies |G(j\omega)| = 0.95,\ \angle G(j\omega) = -90^\circ$$
$$\omega = \sqrt{2} \implies |G(j\omega)| = \frac{1}{3},\ \angle G(j\omega) = -180^\circ$$
$$\omega \to \infty \implies |G(j\omega)| \to 0,\ \angle G(j\omega) = -270^\circ$$



$P = 0$. We can see that if $-\frac{1}{K} \in \left(-\frac{1}{3}, 1\right)$ then $N > 0$ and closed-loop system would be unstable for stability we need $-1 < K < 3$ so that $N = 0$