Reading: FPE, Sections 6.4-6.6, 6.7.1-6.7.6

Problems:

1. Consider the transfer function

\[ G(s) = \frac{s + 1}{s^2 + s + 1} \]

Use MATLAB to compare the \( M_p \) from the step response of the system for \( a = 0.01, 0.1, 1, 10, \) and 100 with the \( M_r \) from the frequency response for the same values of \( a \). Is there a correlation between \( M_p \) and \( M_r \)?

2. Consider the transfer function

\[ G(s) = \frac{1}{(s^2 + 1)(s^2 + s + 1)} \]

Draw the Bode plots for \( p = 0.01, 0.1, 1, 10, \) and 100. What conclusions can you draw about the effect of the pole at \(-p\) on the bandwidth compared with the bandwidth for the second-order system without this pole? MATLAB use is allowed.

3. Consider the following problem: for the system \( G(s) = \frac{1}{s^2} \), design a lead controller that gives \( \text{PM} \approx 90^\circ \) and \( \omega_{BW} \approx 0.5 \). This homework problem asks you to check and improve the design given in class.
   
   a) For the controller:

\[ KD(s) = \frac{1}{16} \frac{s + 1}{s^2 + 1} \]

compute the PM, open-loop crossover frequency \( \omega_c \), and closed-loop bandwidth \( \omega_{BW} \). Plot the closed-loop step response. Explain the reasons why this design didn’t fully meet the specs.

b) Improve the design to obtain PM and \( \omega_{BW} \) closer to the specs. Does the new closed-loop step response show better damping?

4. For the two plant transfer functions given below, use the Nyquist stability criterion to determine all values of the feedback gain \( K \) that stabilize the closed-loop system.

   a) \( G(s) = \frac{1}{(s + 2)(s + 4)} \)

   b) \( G(s) = \frac{1}{(s + 1)(s^2 + s + 1)} \)

Instructions: To draw the Nyquist plot, use the Bode plots of \( G(s) \). Explain all steps in arriving at the Nyquist plot. Start with hand-sketched Bode plots. You can then generate more accurate Bode plots with MATLAB to get exact numerical values if necessary. The use of MATLAB for drawing the Nyquist plot is not allowed (except to check your work at the end). It is also recommended that you check your results with Routh-Hurwitz stability criterion.