ECE 486, Fall 2017 — PROBLEM SET 5 — Due Tuesday, Oct 24

Reading: FPE, Sections 5.3, 5.4.1, 5.4.2, 5.5.

Problems:

1. Consider the transfer function $L(s) = \dfrac{s^2+2s+2}{s^2-2s+2}$.

a) Plot *by hand* the *positive* ($K > 0$) root locus for $L(s)$, using all the rules seen in class. Make your root locus as explicit as possible by specifying (when applicable) the real-axis part, asymptotes, arrival and departure angles, imaginary-axis crossings, and points of multiple roots. Turn in the hand plot and accompanying calculations and explanations.

b) Plot the same root locus in MATLAB. Turn in the MATLAB plot.

2. Consider the plant transfer function $G(s) = \dfrac{1}{s^2-1}$. Suppose that the plant is connected in standard feedback configuration with the lead controller $D_{\text{lead}}(s) = K\dfrac{s+z_{\text{lead}}}{s+p_{\text{lead}}}$ and that the control objective is to place closed-loop poles at $-2 \pm 2j$. We assume throughout that the value of the lead pole is fixed at $p_{\text{lead}} = 10$.

a) Using the root locus phase condition, compute the value of the lead zero that achieves the pole placement objective. (If performing angle computations in MATLAB, note that MATLAB works with radians.)

b) Using the value of z_{lead} that you computed in part a), write down the equation for the gain K that corresponds to the closed-loop poles at $-2 \pm 2j$. Solve it for K. (Hint: you have two equations, one from the real part and the other from the imaginary part, and both should give the same K.)

c) Using MATLAB, plot the root locus for $1 + K\dfrac{s+z_{\text{lead}}}{s+10}\dfrac{1}{s^2-1} = 0$ with the value of z_{lead} found in part a). By mouse-clicking, verify that the value of K you found in part b) indeed corresponds to the desired poles.

d) Is the closed-loop system with these values of z_{lead} and K stable? Explain using the root locus.

e) Find the point of multiple roots on the locus by numerically solving the equation given by Rule 6. Verify the result by mouse-clicking on the locus.

f) With z_{lead} and K found above, compute the closed-loop steady-state tracking error to the unit step reference using the formula $e(\infty) = 1/(1 + D_{\text{lead}}(0)G(0))$. Express the result in percents.

g) Write down the closed-loop transfer function $\dfrac{D_{\text{lead}}(s)G(s)}{1+D_{\text{lead}}(s)G(s)}$. Compute its DC gain. Use the DC gain to find the steady-state tracking error and verify that your result agrees with the one from part f).

h) Using MATLAB, plot the step response of the closed-loop transfer function from part g). Check that it is consistent with the result of part g).

To improve tracking, let's add a lag controller in series with the lead controller. That is, consider the controller $D_{\text{lead,lag}}(s) = K\dfrac{s+z_{\text{lead}}}{s+10}\cdot\dfrac{s+z_{\text{lag}}}{s+p_{\text{lag}}}$, where z_{lead} and K are as before while z_{lag} and p_{lag} are to be chosen.

i) How large does the ratio $z_{\text{lag}}/p_{\text{lag}}$ need to be so that the closed-loop steady-state tracking error to the unit step reference is less than 5%? Use the formula $e(\infty) = 1/(1 + D_{\text{lead,lag}}(0)G(0))$.

j) Fix some values of z_{lag} and p_{lag} which satisfy the condition from part i). Write down the closed-loop transfer function $\dfrac{D_{\text{lead,lag}}(s)G(s)}{1+D_{\text{lead,lag}}(s)G(s)}$. Compute its DC gain and verify that it guarantees tracking within 5%.

k) Using MATLAB, plot the step response of the closed-loop transfer function from part j). Check that the tracking error is improved as predicted in parts i) and j).

l) Using MATLAB, plot the root locus for $1 + K\dfrac{s+z_{\text{lead}}}{s+10}\dfrac{s+z_{\text{lag}}}{s+p_{\text{lag}}}\dfrac{1}{s^2-1} = 0$ with the lead zero, lag zero, and lag pole as above. Do we still get poles at approximately $-2 \pm 2j$ for the same gain value K? If not, go back and make adjustments to the values of z_{lag} and p_{lag}, making sure that the ratio $z_{\text{lag}}/p_{\text{lag}}$ still satisfies the condition of part i). (Hint: the lag zero and lag pole need to be very close to the origin, so as not to interfere with the lead zero and lead pole.) Is the system stable for this value of K? Explain by carefully studying the root locus.

Turn in your hand work as well as MATLAB plots. Accuracy up to the second decimal point is acceptable.

3. Consider the transfer function $G(s) = \dfrac{1}{s^2 + 0.5s + 1}$.

a) Use the formulas given in class:

$$\omega_r = \omega_n\sqrt{1 - 2\zeta^2},\ M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} - 1 \quad (\text{valid for } \zeta < 1/\sqrt{2}),$$

$$\omega_{\mathrm{BW}} = \omega_n\sqrt{(1 - 2\zeta^2) + \sqrt{(1-2\zeta^2)^2 + 1}}$$

(taken from the book by Kuo, Section 9.2) to compute the resonant frequency ω_r, resonant peak M_r, and bandwidth ω_{BW} for $G(j\omega)$.

b) Use a computer to plot the magnitude $|G(j\omega)|$ as a function of ω (you can use the `bode` or `bodemag` command in MATLAB). Mark the resonant frequency ω_r, resonant peak M_r, and bandwidth ω_{BW} on the graph. Check agreement with the values you computed in a).

4. For each of the transfer functions given below, draw the Bode plots (both magnitude and phase) *by hand*, using the techniques discussed in class. Explain all steps in your drawing procedures. Note that the transfer functions are not given in Bode form.

a) $KG(s) = \dfrac{s + 10}{s(s + 5)}$ b) $KG(s) = \dfrac{8s}{s^2 + 0.2s + 4}$ c) $KG(s) = \dfrac{s^2 + 0.1s + 1}{s(s + 0.2)(s + 4)}$

After you're done, check your results using MATLAB. (Note that the `bode` command in MATLAB plots magnitude in decibels.) Turn in both the hand sketches and the MATLAB plots.