ECE 486, Fall 2017 **PROBLEM SET 4** **Due Thursday, *Oct 12***

(one week after the midterm)

Reading: FPE, Sections 5.1, 5.2, 5.6.1.

Problems:

1. The goal of this exercise is to compare sensitivity of open-loop and closed-loop control with respect to errors in the *control gain*. Consider the DC motor model discussed in class, with no disturbance ($w_\ell = 0$). Let the control gain sensitivity be defined as follows: when the controller gain changes from K to $K + \delta K$ and, as a result, the steady state gain (DC gain) of the overall system changes from T to $T + \delta T$, we define $S_K = \dfrac{\delta T/T}{\delta K/K}$. (The motor gain A remains fixed here.)

a) Compute the sensitivity S_K in the open-loop case, starting from the nominal values $K_{ol} = 1/A$ and $T_{ol} = 1$.

b) Compute the sensitivity S_K for a feedback gain K_{cl}, using the approximate formula $\delta T = \dfrac{dT}{dK}\delta K$ and the fact that the nominal system gain is, as derived in class, $T_{cl} = \dfrac{AK_{cl}}{1 + AK_{cl}}$.

Hint: your final answers in a) and b) should be the same as the ones derived in class for sensitivity to errors in the motor gain A.

2. Suppose that the DC motor discussed in class is connected in feedback with a PI controller $k_P + k_I/s$. (This refers to the standard feedback control configuration, where the input to the controller is $e = r - y = \omega_{ref} - \omega_m$.) Write down the full transfer function of the closed-loop system in the presence of load/disturbance w_ℓ. (For $k_I = 0$ this should match what we derived in class for constant feedback gain.) Is it true that by proper choice of gains k_P and k_I we can achieve arbitrary pole placement as well as perfect constant reference tracking and constant disturbance rejection in steady state? Justify your answer.

3. Consider the plant with transfer function $L(s) = \dfrac{1}{s^2 + 2s}$. Under the action of a constant feedback gain K, the closed-loop poles are the roots of the characteristic polynomial $s^2 + 2s + K$.

a) Draw the (positive) root locus. (Use the expression for the closed-loop poles in terms of K obtained via the quadratic formula.)

b) Consider the settling time spec $t_s \leq 4$. Give some value (or range of values) of K for which the closed-loop system meets this spec. Justify your choice. Show the corresponding pole locations on the root locus.

c) Consider the rise time spec $t_r \leq 1$. Give some value (or range of values) of K for which the closed-loop system meets this spec. Justify your choice. Show the corresponding pole locations on the root locus.

d) Consider the overshoot spec $M_p \leq 0.1$. Give some value (or range of values) of K for which the closed-loop system meets this spec. Justify your choice. Show the corresponding pole locations on the root locus.

e) Suppose that it is desired to place the closed-loop poles at $-1 \pm j$. Find the value of K that will achieve this, using the characteristic equation $s^2 + 2s + K = 0$ *but without using the quadratic formula.* (In other words, you should find a way of doing this that would also work for a higher-order example.)

4. Consider the following transfer functions:

$$1)\ L(s) = \frac{1}{s(s^2+4s+8)} \qquad\qquad 2)\ L(s) = \frac{s}{(s-1)(s+1)^3}$$

For each one of these, do the following:

a) Mark the zeros and poles on the s-plane and plot the real-axis part of the root locus.

b) Use the phase condition from class to test whether or not the point $s = j$ is on the root locus. If you run into "non-obvious" angles, *estimate* rather than *calculate* them, this should be enough.

c) Determine asymptotes and departure and arrival angles. Plot the root locus branches based on this information.

d) Determine imaginary-axis crossings (if any), and complete the (positive) root locus by using Rule 6 to check for multiple roots.

e) Plot the (positive) root locus using the MATLAB `rlocus` command.

Turn in your MATLAB plots as well as hand sketches of root loci along with all accompanying calculations and explanations.