ECE 486 **Assignment # 3**

Issued: September 14 Due: September 21, 2017

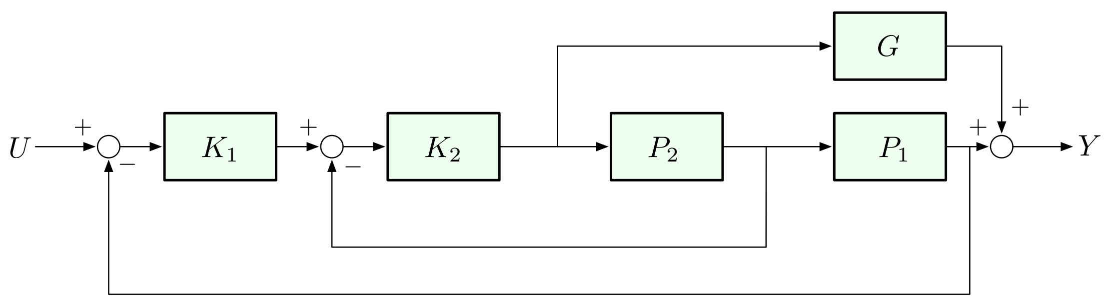
Reading Assignment:

FPE, Sections 3.3-3.6.

Problems:

(unless otherwise noted, you can use a calculator/computer to arrive at numerical answers)

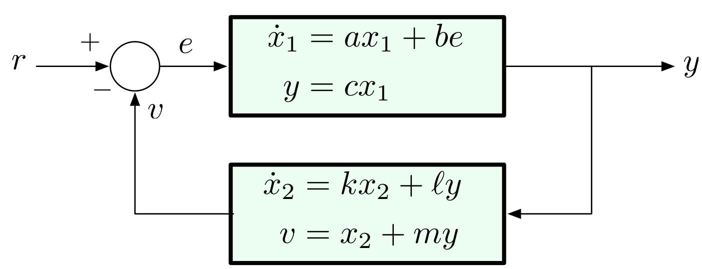
1. Consider the system given by the block diagram below:



   Compute the transfer function from the input U to the output Y.

   (Your answer should be an expression involving the transfer functions K_1, K_2, P_1, P_2, G.)

2. Consider two systems in a negative feedback configuration shown below:



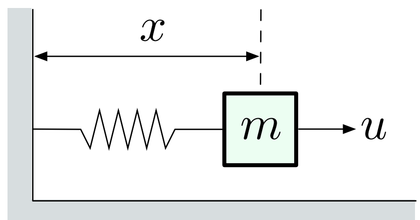
   All state variables, all inputs, and all outputs are scalars.

   (i) Find the transfer function from the input R to the output Y.

   (Your answer should be a ratio of two polynomials in s with coefficients expressed in terms of the system parameters a, b, c, k, ℓ, m.)

   (ii) Write down the conditions that must be satisfied by the system parameters a, b, c, k, ℓ, m for this transfer function to be stable (i.e., for all poles to have negative real parts).

3. Consider the mass-spring system shown in the diagram below:



As shown in class, this system has the following state-space model with $x_1 = x$ and $x_2 = \dot{x}$:

$$\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\rho}{m} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix} u, \qquad y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},$$

where k is the spring constant and ρ is the friction coefficient.

(i) Determine the transfer function of this system.

(ii) Now suppose that the output equation is replaced by

$$y = \begin{pmatrix} c_1 & c_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},$$

where c_1 and c_2 are arbitrary constants. Find the values of c_1 and c_2 to guarantee that the transfer function of the resulting system has the form of the prototype 2nd-order response discussed in class. Write down the expressions for the parameters ω_n and ζ in terms of k, ρ, and m.

(iii) Determine the steady-state response of the system from part (ii) to the sinusoidal external force $u(t) = \cos(\omega t)$.

(iv) Sketch the plots of the magnitude and the phase shift of the steady-state response from part (iii) as functions of the input frequency ω. What happens when the input frequency ω matches the system's natural frequency ω_n? This is the phenomenon known as *resonance*.

4. Consider the transfer function

$$H(s) = \frac{16}{s^2 + 4s + 16}$$

(i) Suppose that you are given the following time-domain specs: $t_r \leq 0.6, t_s \leq 1.6$ (where $t_s = t_s^{5\%}$). Plot the admissible pole locations in the s-plane corresponding to these two specs. Does the given system satisfy these specs?

(ii) Suppose that in addition to the specs from (i), we have the following spec on the overshoot: $M_p \leq 1/e^2$. Plot the admissible pole locations in the s-plane corresponding to all three specs. Does the given system satisfy the new spec?

(iii) Now suppose that you are given the two specs from (i) *plus* the following spec on the peak time: $t_p \leq 1$ (*instead of* the overshoot spec). Plot the admissible pole locations in the s-plane corresponding to these three specs. (Pole locations for peak time were not discussed in class, so you need to derive this.) Does the given system satisfy the new spec?