ECE 486, Fall 2017 **PROBLEM SET 2** **Due Thursday, Sep 14**

Reading: FPE, Sections 3.1 and 3.2.

Problems:

1. a) Derive *by hand* the Laplace transform of $2\sin(2t)$. Hint: use Euler's formula. Check your answer in the Laplace transform tables.

b) As derived in class, the (steady-state) frequency response of the system with transfer function $H(s)$ to the signal $A\cos(\omega t)$ is $AM\cos(\omega t + \phi)$, where $H(j\omega) = Me^{j\phi}$. Do a similar calculation to derive the steady-state response to $A\sin(\omega t)$.

2. Consider the system with transfer function $H(s) = \dfrac{1}{s+2}$. Assume that the initial condition is zero.

a) Compute the response to the input $u(t) = 2\sin(2t)$ by applying the formula $Y(s) = H(s)U(s)$ with $U(s)$ from problem 1(a), then using partial fractions, and finally Laplace transform tables. You may find the MATLAB command `residue` helpful for checking the results of your partial fractions calculation (but you must derive them by hand and show your work).

b) Compute the response to the same input by using the frequency response formula you obtained in problem 1(b). What is the difference between the answers in part a) and part b), and how is it related to the pole location of the transfer function?

3. Consider the following transfer functions:

$$H_1(s) = \frac{2}{s^2 - 2s + 4}, \qquad H_2(s) = \frac{2s-3}{s^2+4s+1}.$$

a) Use the Final Value Theorem to compute their DC gains.

b) Use the MATLAB command `step` to plot their step responses. (You may also find the command `ltiview` convenient to use for such tasks.) Submit your plots.

c) In each case, explain whether the Final Value Theorem gives the right result, and why.

4. Consider the following state-space model (so-called "observer canonical form"):

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & -a_0 \\ 1 & -a_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} b_0 \\ b_1 \end{pmatrix} u, \qquad y = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

a) Show that its transfer function is $H(s) = \dfrac{b_1 s + b_0}{s^2 + a_1 s + a_0}$.

Hint: write out the differential equations, then switch to the s-domain using the differentiation rule for Laplace transforms, and use the resulting equations to solve for $Y(s)$ in terms of $U(s)$.

b) Build an all-integrator diagram for this system.

This will be covered in class on Tuesday, but your diagram will be different from the all-integrator diagram given in class for the system in "controller canonical form," even though the two systems have the same transfer function. But you should be able to see how the two diagrams are related (loosely speaking, they are "mirror images" of one another, with summer junctions and splitters interchanged).