we have analyzed the response of first and second order systems. we can now think of designing feedback controllers to shape the response of such systems. we consider the basic closed loop system below:

we include the power amplifier and actuator as part of the plant and ignore the sensor dynamics, for simplicity.
we consider three basic types of control action

- Proportional
- Derivative
- Integral

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A proportional control produces a signal proportional to the error \( E(s) = R(s) - C(s) \), i.e.

\[
U(s) = K_p E(s)
\]

where \( K_p \) is the proportional gain of the compensator.

The transfer function of the compensator is simply

\[
H(s) = \frac{U(s)}{E(s)} = K_p
\]
A Derivative control is a control proportional to the derivative of the error.

\[ U(s) = K_D E'(s) \]

The transfer function is

\[ H(s) = \frac{U(s)}{E(s)} = K_D s \]

A Integral control is a control proportional to the integral of the error.

\[ U(s) = \frac{K_I}{s} E(s) \]

The transfer function is

\[ H(s) = \frac{K_I}{s} \]
usually a combination of proportional, integral, and derivative control actions are required to achieve good performance.

Proportional + Integral (PI)

\[ H(s) = K_p + \frac{K_i}{s} = K_p \left(1 + \frac{1}{T_i s}\right) \]

\( T_i \) is called the integral time.

Proportional + Derivative (PD)

\[ H(s) = K_p + K_d s = K_p \left(1 + T_d s\right) \]

\( T_d \) is called the derivative time.

The most general type is the (PID) Proportional + Integral + Derivative Control.
\[ H(s) = K_p + K_d s + K_i / s \]

\[ = K_p \left( 1 + \frac{1}{T_i s} + T_d s \right) \]

\[ = K_p \left( \frac{T_i T_d s^2 + T_i s + 1}{T_i s} \right) \]

PID control is the most widely used controller in industry. The relative effect of the various terms depends on the choice of gains. In fact, the design problem is to 'tune' the gains \( K_p, K_d, K_i \), or equivalently \( K_p, T_d, T_i \).
Examples

1) Satellite Orientation Control

Consider a satellite equipped with gas jets to control the orientation about the vertical axis. If the rotational inertia about this axis is \( J \) and the gas jets produce a tangential torque \( \tau \), we can model the plant as

\[ J \ddot{\theta} = \tau \]
The open loop system is

\[ T(s) \rightarrow \frac{1}{Js^2} \rightarrow \Theta(s) \]

If we choose a PD control

\[ T(s) = \left( K_p + K_0 s \right) E(s) \]

we have

\[ \Theta_d \rightarrow E \rightarrow K_p + K_0 S \rightarrow \frac{1}{Js^2} \rightarrow \Theta \]

The closed loop transfer function is then

\[ T(s) = \frac{\Theta(s)}{\Theta_d(s)} = \frac{K_p + K_0 S}{Js^2} \times \frac{1}{1 + \frac{K_p + K_0 S}{Js^2}} = \frac{K_p + K_0 S}{Js^2 + K_0 S + K_p} \]
Divide through by $J$ and write

$$T(s) = \frac{K_p/J + K_0/J s}{s^2 + K_0/J s + K_p/J}$$

The closed loop characteristic polynomial is $s^2 + K_0/J s + K_p/J$

Given a desired natural frequency and damping ratio, $\omega_n$, so we can write

$$s^2 + \frac{K_0}{J} s + \frac{K_p}{J} = s^2 + 2\zeta \omega_n s + \omega_n^2$$

$$\Rightarrow K_p = J\omega_n^2, \quad K_0 = 2JS\omega_n$$

The particular $S$, $\omega_n$ would be chosen from the design specs - desired overshoot, rise time, settling time, etc.
Since the satellite is likely to be quite massive and the gas jets capable of producing only a small torque, we would not be able to specify a fast response. For example, with \( w = 0.1 \), \( \xi = 1 \), \( J = 100 \) we have \( K_p = 2 \), \( K_0 = 20 \). The step response is shown below.
we note the effect of the zero at $s = -\frac{K_p}{K_0}$ which results in a faster response and overshoot. We can see the reason for this as follows:

The output $\Theta(s)$ is given by

$$\Theta(s) = \frac{K_p/j + K_0/j \cdot s}{s^2 + K_0/j \cdot s + K_p/j} \Theta_d$$

$$= \frac{K_p/j}{s^2 + K_0/j \cdot s + K_p/j} \Theta_d$$

$$+ \frac{K_0/j \cdot s}{s^2 + K_0/j \cdot s + K_p/j} \Theta_d$$

The first term produces the predicted rise time, overshoot, etc. based on $S$, $\omega_n$. 
The second term involves $5\theta$. If $\theta$ is the unit step, then an impulse is generated. This impulse response is superimposed on the first term.
an alternate implementation
of a PD-control is shown
below

\[ \Theta_d \rightarrow \Theta \]

This means that

\[ Z(s) = K_p E(s) - K_d S \Theta(s) \] and avoids differentiating the discontinuity in the step reference input \( \Theta_d \).

What is the closed loop transfer function in this case?
Exercise: Show that the closed loop transfer function is
\[ T(s) = \frac{\Theta(s)}{\Theta_d(s)} = \frac{\frac{K_p}{J}}{s + \frac{K_p}{J}} \]

Remarks: This implementation is known as a rate feedback control or two degree of freedom control. It is particularly useful when the rate \( \frac{d\Theta}{dt} \) is measured directly with a sensor. In this case no differentiation is required by the compensator.
Exercise: Show that this is equivalent to the previous block diagram, i.e. show that

\[
\frac{\Theta(s)}{\Theta_d(s)} = \frac{\frac{K_p}{s}}{s^2 + \frac{K_0}{Js} + \frac{K_p}{s}}
\]