ECE 486 Control Systems

Exam # 2

November 7, 2006

Name (Print): Answer Key

Problem 1: (10 Points) ______
Problem 2: (20 Points) ______
Problem 3: (15 Points) ______
Problem 4: (10 Points) ______
Problem 5: (10 Points) ______
Problem 6: (10 Points) ______
Problem 7: (15 Points) ______
Problem 8: (10 Points) ______
Total: (100 Points) ______

INSTRUCTIONS:
The exam is closed book. You are allowed a single $8\frac{1}{2} \times 11$ page of notes. Calculators are permitted. The point total for each problem is indicated. Do all of your work on the exam pages given. Show all your work for partial credit.
GOOD LUCK!
1. Match the root locus of $1 + KG(s)$ to the corresponding transfer function.

   a) $G(s) = \frac{1}{s(s + 1)(s + 2)}$

   b) $G(s) = \frac{(s - 1)(s - 2)}{s(s + 3)(s + 2)}$

   c) $G(s) = \frac{s + 2}{s(s + 1)(s + 4)}$

   d) $G(s) = \frac{s + 2}{s(s + 1)(s^2 + 2s + 2)}$

   Root Locus = \begin{cases} 2 \end{cases}

   Root Locus = \begin{cases} 5 \end{cases}

   Root Locus = \begin{cases} 7 \end{cases}

   Root Locus = \begin{cases} 1 \end{cases}
2. Consider the transfer function

\[
G(s) = \frac{1}{(s - 1)(s + 3)(s + 5)}
\]

Fill in the information below and sketch the root locus of \(1 + KG(s) = 0\) for

a) Angle of asymptotes 
\[
\phi = \frac{180 + 360(L-1)}{n-m}, \quad L = 1, ..., n-m; \quad n = 3, m = 0
\]

\[
= 60^\circ, 180^\circ, 300^\circ
\]

b) Centroid of asymptotes 
\[
a = \frac{\sum P_i - \sum R_i}{n-m} = \frac{1-3-5}{3} = -\frac{7}{3}
\]

c) Range of \(K\) for stability 

The characteristic polynomial is
\[
(s-1)(s+3)(s+5) + K = s^3 + 5s^2 + 7s + K - 15
\]

\[
\text{for stability we must have } \quad 6K > K > 15
\]
3. (a) To the transfer function \( G(s) = \frac{1}{(s - 1)(s + 3)(s + 5)} \) from the previous problem add a zero at \( s = -6 \). Sketch the root locus and find the range of \( K \) for which the closed loop system is stable.

\[
\phi = \frac{180 + 360(k-1)}{n-m}, \quad \rho = 1, \ldots, n-m ; \quad n=3, \quad m=1
\]

\[
\phi = 90, 180
\]

\[
a = \frac{1 - 3 - 5 - (-6)}{2} = -\frac{1}{2}
\]

The characteristic polynomial is

\[
s^3 + 7s^2 + 15s - 15 + K(s+6) = s^3 + 7s^2 + (7+k)s + 6K-15
\]

\( \Rightarrow \) stable for \( K > \frac{15}{6} \)

In this case
4. Sketch the asymptotic Bode magnitude plot of each of the following transfer functions.

a). \( G(s) = \frac{160}{(s+4)(s+40)} = \frac{1}{(\frac{s}{4} + 1)(\frac{s}{40} + 1)} \)

b). \( G(s) = \frac{10}{s(s+10)} = \frac{1}{s \left( \frac{s}{10} + 1 \right)} \)
5. Consider the Bode plot shown below.

(a) Why does the phase plot start at $-270^\circ$ at low frequencies? (Circle one)
   i. The open-loop system is unstable.
      ii. There is a pole at $s = -270$.
      iii. There is a zero at $s = -270$.
      iv. The phase margin is infinite.

(b) Is the closed-loop system stable? \textbf{YES}

(c) Will the closed-loop system be stable if the gain is lowered by a factor of 100? Justify your answer.
   \textbf{NO; phase $<-180^\circ$}
6. Classify the following compensators as PD, PI, PID, Lead, or Lag.

   a) \( G_c(s) = \frac{s + 4}{s + 6} \)  \[ \text{Lead} \]

   b) \( G_c(s) = \frac{0.1s + 3}{s} \)  \[ \text{PI} \]

   c) \( G_c(s) = \frac{s + 0.4}{s + 0.3} \)  \[ \text{LAG} \]

   d) \( G_c(s) = \frac{5s^2 + 4s + 1}{s} \)  \[ \text{PID} \]
7. Consider the unity feedback system below.

\[
\begin{array}{ccc}
R(s) & \xrightarrow{G_p(s)} & G_c(s) & \xrightarrow{G_l(s)} & C(s)
\end{array}
\]

With \(G_p(s) = \frac{1}{s(s+2)}\) design a Lead compensator, \(G_c(s)\), so that the closed loop system has a pair of poles at \(s = -4\). [Hint: try a pole-zero cancellation.]

With \(G_c(s) = K \frac{s+2}{s+p}\), try \(p = 2\), i.e.

a pole-zero cancellation. Then

\[
1 + K G_c(s) G_p(s) = 1 + K \frac{1}{s(s+p)} = 0
\]

In order that the system have a pair of poles at \(s = -4\), choose \(p = 8\). Then the characteristic polynomial is

\[
s^2 + 8s + K = (s+4)^2 = s^2 + 8s + 16
\]

\[
\therefore K = 16
\]

The complete compensator is

\[
G_c(s) = 16 \cdot \frac{s+2}{s+8}
\]
8. Consider the Nyquist plot shown below for a given system. Assume that the system has no open-loop unstable poles.

(a) What are the gain and phase margins assuming $\alpha = 0.4$, $\beta = 1.3$, $\phi = 40^\circ$?

(b) Describe what happens as the gain of the closed-loop system goes from zero to a very large value.

(c) Sketch what the root locus near the imaginary axis would look like for such a system.

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a) The phase margin is $\text{PM} = 40^\circ$.

The gain margin is more difficult. In this case if the gain is increased by $\frac{1}{\alpha}$ or decreased by $\beta$, then the system will be unstable. The Bode plot would cross $-180^\circ$ twice.

b) The system would go from stable to unstable to stable and finally to unstable.