PSet 8 Solution.

8.1 a. Use KVL loops to see that the voltage peaks are \( V_{\text{in}, \frac{N_2}{N_1}} \) and \( V_{\text{in}, \frac{N_3}{N_1}} \) due to the voltage polarities activate certain diodes at each portion of the secondary side.

Example: \( q_i(t) = 1 \), \( D_1 \) activates but not \( D_2 \).

There is a voltage across the entire secondary, but this is center-tapped so only a portion of the voltage is balanced at:

\[ V_x \]
\[ V_{\text{in}, \frac{N_2}{N_1}} \]
\[ V_{\text{in}, \frac{N_3}{N_1}} \]

\[ D_1 \]
\[ \frac{I}{2} \]
\[ \frac{I}{2} \]
\[ D_2 \]
\[ I \]
b. Can use \( L \) to find the output voltage:

\[
<V_L> = D_1 (V_{in} \frac{N_{S1}}{N_{P2}} - V_{out}) + D_2 (V_{in} \frac{N_{S2}}{N_{P1}} - V_{out}) - V_{out} (1 - D_1 - D_2) = 0
\]

\[
= D_1 V_{in} \frac{N_{S1}}{N_{P2}} + D_2 V_{in} \frac{N_{S2}}{N_{P1}} - V_{out}
\]

\[
\frac{V_{out}}{V_{in}} = D_1 \frac{N_{S1}}{N_{P2}} + D_2 \frac{N_{S2}}{N_{P1}}
\]

C. \[
\frac{V_{out}}{V_{in}} = D \frac{N_{S}}{N_{P}} + D \frac{N_{S}}{N_{P}} = (2D) \frac{N_{S}}{N_{P}}
\]
8.2.a. Use $L$ at the output to interpret the converter.

For $q(t)=1$:

$$V_L = V_{in} \frac{N_2}{N_1} - V_{out}$$

$q(t)=0$:

$$V_L = -V_{out}$$

$q(t)=1$ is the only time voltage can be supplied during the cycle, so $D_3$ must be on.

$q(t)=0$ has $\frac{1}{M_1}$ cycle in a loop back into $V_{in}$.

$$\frac{V_{out}}{V_{in}} = D \frac{N_2}{N_1}$$

b. $L_{m1} \frac{dI}{dt} = V_{in}$

$$I = \frac{V_{in} \delta}{L_{m1}} = \frac{12}{10^{-4}} \cdot \frac{D_1 \cdot 0.4}{60 \times 10^3} = \frac{12 \cdot 0.4}{60} = 0.8 \text{A}$$

Since $<V_{Lm1}> = 0$, the voltage can be graphed as:

$$\frac{V_{in}}{0.82}$$
The reason it goes to 0 at 0.8 T is so the average voltage is 0.

Since the voltage goes to 0, there must not be any current. This means
                               - \( I_{2m} = 0 \) for diodes to shut off.

\[ I_{2m} \times T = 2 I = 0.8 A \]

\[ I_{2m} = \frac{0.8}{T} \]

If \( 177 V \), then current increases with each cycle and saturates the transformer.

A. \( V_{out} = V_{in} \times \frac{N_2}{N_1} \) Assume \( V_{in} = 12V \)

\[ = 12 \times \frac{N_2}{N_1} = 200 \]

To find a value of \( R \), use conservation of power to find current from \( V_{in} \).
\[ P_{in} = P_{out} = 12 \cdot I_{in} = 200 \quad R = \frac{V^2}{P} = 200 \Omega \]

\[ I_{in} = \frac{50}{3} \text{ A} \quad \Rightarrow I_{out} = 1 \text{ A} \]

\[ I_{out} = I_{in} \frac{N_2}{N_1} D \quad \text{through conservation of power in a transformer.} \]

\[ = \frac{50}{3} \frac{N_2}{N_1} D \]

From before: \[ D \frac{N_2}{N_1} = D \frac{200}{12} = \frac{50}{3} D \]

Can pick any \( D < D \leq 0.5 \)

\[ \Rightarrow \frac{N_2}{N_1} = \frac{50}{3} \quad \text{Must follow these ratios} \]

\[ R = 200 \Omega \]

Ex.: \( D = \frac{1}{3} \quad \Rightarrow \frac{N_2}{N_1} = 50 \)
e. Peak current through $D_1$ would just be 
$$i_1 = \frac{V_{in}}{L_{in}} \cdot \text{DoT} = \frac{12}{10^{-6}} \cdot \frac{1}{6 \cdot 10^{-4}} = 2.1$$

Peak current in $Q_2$ is $I_{in}$ due to current going through transformer.
$\frac{V_{in}}{12 \cdot V} = \frac{50}{3.1}$

$D$ is picked.

$Q_2$ has much higher stress, by orders of magnitude.

Labeling as $\frac{N_2}{N_1} \times 1A$ is also fine. (Since $\frac{N_2}{N_1} = \frac{50}{30}$)
a) \( <V_{L1} > = 0 \)

\[
V_{in} DT + V_{caux}(1 - D)T = 0
\]

\( V_{caux} = -\frac{D}{1 - D} V_{in} \)

b)\[
\begin{array}{c|c}
\text{i}_{L1} & t (\mu s) \\
\hline
-1 & 0 \\
1 & 2 \\
\hline
\end{array}
\]

\( <i_{caux} > = 0 \)

\[
i_{caux} = \begin{cases} 0 & q(t) = 1 \\ -i_{L1} & q(t) = 0 \\ \end{cases}
\]

\( <i_{L1} > q(t) = 0 = 0 \)

\( \Delta i_{L1} = \frac{V_{in}}{L_{L1}} DT = 2A \)

\( i_{L1} \) has both positive and negative region

\( \) the transformer has bipolar flux swing

c) the transformer should be wound on an ungapped core

\( L_{L1} = \frac{N_{1}^{2}}{R_{c}} \)

ungapped core \( R_{c} \ll \) gapped core

\( L_{L1} \) is maximized \( i_{L1, pk} \propto \frac{1}{L_{L1}} \) is minimized

\( \frac{1}{2} L_{L1} i_{L1}^{2}, pk \) is minimized.
d) \[ V_{\text{out}} = \frac{N_2}{N_1} \Delta V_{\text{in}} = \frac{40}{10} \times 0.4 \times 50 = 80 \text{ (V)} \]

\[ I_{\text{out}} = \frac{V_{\text{out}}}{R} = 8 \text{ (A)} \]

\[ q(t) = 1 \]

\[ I_{\text{sw, pk}} = \frac{N_2}{N_1} I_{\text{out}} + I_{\text{m, pk}} \]

\[ = 4 \times 8 + 1 = 33 \text{ (A)} \]

\[ q(t) = 0 \]

\[ V_{\text{sw, pk}} = V_{\text{in}} - V_{\text{cmax}} \]

\[ = 50 + \frac{0.4}{0.6} \times 50 = 83.3 \text{ (V)} \]

2) Voltage rating \[ > 83.3 \times 2 = 166.6 \text{ V} \]

Current rating \[ > 33 \times 2 = 66 \text{ A} \]

We can choose the \textit{N-Channel MOSFET} part No. IRFS4127TRLPBF from Infineon

with rated voltage \( V_{dss} = 200 \text{V} \)

rated current \( I_d = 7 \text{A} \)

The unit price is \$2.65/2 \text{ for } 1 \text{K quantities (digikey.com)}
8.4 a. Using KVL loops, it can be seen 
\[ q_1 = q_2 = \frac{1}{\pi} \] for \( V_0 = V_i \) and \( q_3 = q_4 = 1 \) for \( V_0 = -V_i \).
To allow current to circulate when \( V_0 = 0 \),
\[ q_1 = q_4 = 1 \] or \( q_2 = q_3 = 1 \).

b. Fourier Expansion:
\[ V_0(t) = q_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t) \]

\( a_n = 0 \) because there is no DC offset
\( b_n = 0 \) because the wave resembles \( \sin \).

\[ b_n = \frac{2}{\pi} \int_{0}^{\pi} V_0(t) \sin(n\omega t) dt \]
\[ = \frac{2}{\pi} \left[ \int_{0}^{\pi/2} V_i \sin(n\omega t) dt \right] = \frac{2}{\pi} \left[ V_i \left( \frac{1}{n} \right) \left( 1 - \cos \left( \frac{n\pi}{2} \right) \right) \right] \]
\[ = \frac{V_i}{n} \left( 1 - \cos \left( \frac{n\pi}{2} \right) \right) \]
\[ V_i \left[ \frac{1}{n \pi} \right] = \frac{1}{n \pi} V_i \left[ \frac{y \cos(n \delta)}{n \pi} \right] \]

Yields 0 when \( n = \text{even} \)

\[ \Rightarrow -\cos(n \delta) = \cos(n \delta) \]

\[ -\cos(n \delta) = \cos(n \delta) \]

\[ \cos(n(2\pi - \delta)) = \cos(\delta) \]

\[ \Rightarrow \]

\[ V_U(\delta) = \sum_{n=1}^{\infty} \frac{y \cos(n \delta)}{n \pi} \sin(n \beta) \]

\( n = \text{odd} \)
c. for each frequency \( n \cdot \omega \)

\[ \overline{I_a,n} = \frac{V_{0,n}}{Z_n} \quad \text{(phasor)} \]

\[ Z_n = R + j n \omega L = \sqrt{R^2 + (n \omega L)^2} \quad \angle \tan^{-1} \left( \frac{n \omega L}{R} \right) \]

\[ i_a = \sum_{n=1}^{\infty} \frac{4V_i \cos(n \delta)}{n \pi \sqrt{R^2 + (n \omega L)^2}} \sin \left( n \omega t - \tan^{-1} \left( \frac{n \omega L}{R} \right) \right) \]

\[ P_{R,1} = I_{a,1,\text{rms}} R \]

\[ = \left( \frac{4V_i \cos \delta}{\pi \sqrt{R^2 + \omega^2 L^2}} \right)^2 \frac{1}{\sqrt{2}} R \]

\[ = \frac{8V_i^2 \cos^2 \delta}{\pi^2 (R^2 + \omega^2 L^2)} R \]

e. \quad P_R = \frac{1}{T} \int_0^T i_a(t) R \, dt = \frac{1}{T} \int_0^T \left( \sum_{n=1}^{\infty} i_a,n(t) \right)^2 R \, dt \]

\[ = \sum_{n=1}^{\infty} \frac{1}{T} \int_0^T i_a,n(t) R \, dt \quad \text{(Using the hint, cross multiplication terms with different frequencies are 0)} \]

\[ = \sum_{n=1}^{\infty} I_{a,n,\text{rms}} R \]

\[ = \sum_{n=1}^{\infty} \left( \frac{4V_i \cos(n \delta)}{n \pi \sqrt{R^2 + (n \omega L)^2}} \right)^2 \left( \frac{1}{\sqrt{2}} \right)^2 R \]

\[ = \left( \sum_{n=1}^{\infty} \frac{8V_i^2 \cos^2(n \delta)}{n^2 \pi^2 (R^2 + n \omega L)^2} \right) R \]

f. \( \delta = 30^\circ \)

So \( \cos(3\delta) = 0 \), \( V_{0,3} = 0 \).
a) The converter effectively looks like

340 V DC

So it is a forward converter topology.

RC clamp circuit is used. (as shown by R16, R17, C16).

b) \[ V_{out} = \frac{N_2}{N_1} \cdot D \cdot V_{in} \]

for \( V_{in} = 340 \), \( V_{out} = 5 \), \( \frac{N_2}{N_1} = \frac{1}{16} \),
\( D = 0.235 \)

c) One way is shown below.

Other ways with correct dot placement and winding method are also fine.