Motivation: This problem set is designed to expose you to the modeling and analysis for transformers, often used in power electronics for isolation. Also, you will use these transformers to understand and explore isolated dc-dc converters.

Problem 7.1
(a) Place the missing dots on the transformers terminals shown in Figure 7.1(a). [Hint: Watch out! we have deliberately put voltage polarities and current directions without sticking to the convention discussed in class].

Figure 7.1(a): Transformers

(b) Place the missing winding on the transformer shown in Figure 7.1(b) to agree with the dot placements.

Figure 7.1(b): Another transformer with a missing winding
**Problem 7.2**

Three friends Nikola (Tesla), Thomas (Edison), and William (Stanley) went for an internship at Orange Inc. They were assigned to design, build, and model a two-winding transformer for a new isolated dc-dc converter. After designing and building the transformer—because of intense workload—Nikola fell sick and had to take time-off. Thomas and William were assigned to find out the parameters for the equivalent circuit of the transformer. Thomas used short-circuit and open circuit tests to measure the inductances of the transformer and found out the following values: \( L_{11} = 10.1 \text{ nH} \), \( L_{22} = 44 \text{ nH} \), and \( M = 10.8 \text{ nH} \). He concluded that a transformer can be represented as a magnetically coupled-circuit given by

\[
\begin{bmatrix}
    v_1 \\
    v_2
\end{bmatrix} = \begin{bmatrix}
    L_{11} & M \\
    M & L_{22}
\end{bmatrix} \frac{d}{dt} \begin{bmatrix}
    i_1 \\
    i_2
\end{bmatrix}
\]

The three inductances he measured were sufficient to find the equivalent circuit model of the transformer.

(a) As Thomas took ECE 464, he used the measured values to find the parameters of the equivalent circuit of the transformer given in Fig. 7.2(a). However, he did not know the actual turns-ratio that Nikola used. He assumed the turns-ratio to be unity. Help Thomas compute \( L_{lk1} \), \( L_m \), and \( L_{lk2} \), based on the assumption of unity turns ratio (i.e. \( N_1:N_2 = 1:1 \)).

(b) As William did not take ECE 464, he searched for IEEE papers for an equivalent circuit of a transformer and found one**, shown in Fig. 7.2(b). Help William compute \( L_x \), \( L_y \), and \( n \).

(c) After recovering from his illness, Nikola came back. Since he knew the physical turns ratio (after all he designed it!), he used the measured data to find the parameters of the well-known equivalent circuit given in Fig. 7.2(c). The physical turns-ratio was in fact 1:3. Help Nikola compute \( L_{lk1} \),

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![Fig 7.2 Different equivalent circuit models for a transformer.](image)
L_m, and L_{lk2}.

(d) After finding the parameters of their individual models, they all went to their Manager. Manager was not at all happy and was astonished to find that one physical system can have three equivalent circuits! Who do you think has the correct equivalent circuit model of the transformer and corresponding parameters—Nikola, Thomas, or William? Why?


**Problem 7.3**

Figure 7.3 shows a tapped-inductor boost converter. This approach is sometimes used to achieve larger conversion ratios as compared to a conventional boost design. The inductor winding contains a total of \((N_1+N_2)\) turns. The switch is placed \(N_1\) turns from the left side of the inductor, as shown. The tapped inductor can be viewed as a two-winding \((N_1:N_2)\) transformer, in which the two windings are connected in series. The inductance of the entire \((N_1+N_2)\) turn winding is \(L\).

a. Sketch an equivalent circuit model for the tapped inductor that includes a magnetizing inductance and an ideal transformer. Label the values of the magnetizing inductance and turns ratio.
b. Derive an analytical expression for the conversion ratio \(V_2/V_1\) assuming that all components are lossless, and that the converter operates in continuous conduction mode.
c. Plot the conversion ratio as a function of duty ratio \(D\) for the case \(N_1=N_2\), and compare to the conventional non-tapped case \((N_2 = 0)\).

![Fig 7.3: A tapped-inductor boost converter.](image)

**Problem 7.4**

Shown in Figure 7.4 is a very bad way to connect a transformer, which involves a 12 V battery. You may assume that the battery is ideal (i.e., zero internal resistance). The transformer has 10 turns on the primary winding, a saturation flux density of 1.2 T, a core cross-sectional area \(A_c\) of \(10^{-4}\) m², and a relative permeability of 100 (i.e., \(\mu_r=100\)). You may assume that the saturation characteristics of the transformer is ideally hard, or in other words, the relative permeability of the core material is 100 for \(B < B_{sat}\), and 1 for \(B > B_{sat}\). Moreover, the magnetizing inductance (as measured from the primary side) is 12 \(\mu H\).

a) At what time does the transformer saturate? Please provide a numerical answer.
b) Find the time at which the primary current reaches 200 A. Please provide a numerical answer.
Problem 7.5
Consider the isolated SEPIC converter of Fig. 7.5. You may assume that the transistor and diode are ideal, and that the voltage ripples on all capacitors are small. You may also assume that the inductor Lin is large, such that its ripple can be ignored. The transistor is on with duty ratio D.

(a) Calculate the voltage $V_t$ in periodic steady state.

(b) Derive the input to output conversion ratio with a transformer turns ratio of $1:n$?

(c) Now consider the transistor stress $V_{sw,pk}$ and $I_{sw,pk}$. Find an expression for $V_{sw,pk}$ in terms of $V_{in}$ and $D$, and an expression for $I_{sw,pk}$ in terms of $I_{in}$ and $D$. (Here you may assume that the converter is operating in heavy CCM, and that you can ignore the effects of current ripple in the magnetizing inductance.)

(d) If the nominal input voltage is 20 V, and the desired output voltage is 100 V, what is the optimum transformer turns ratio to minimize the total transistor stress, as defined earlier $S_T = V_{sw,pk}I_{sw,pk}$? At what nominal duty ratio would the converter operate in this case?

(e) Carefully sketch the currents $i_t$, $i_u$, and $i_D$ for an input voltage of 20 V, a turns ratio $N_2:N_1 = 20:5$, and a duty cycle of 0.5, with a load resistor of 40 Ω, a magnetizing inductance of 50 μH, and a switching frequency of 100 kHz. Note that for this part of the question you may not ignore the ripple in the magnetizing inductance. Be sure to provide numerical values for all maximum and minimum values of the currents.

(f) Would it be better to build the transformer with a gapped core or an ungapped core? Why?

![Transformer model](image)