6.1a. Ni here serves as the source of the flux, and there are two reluctances here: the core and air gap. These will be $R_c$ and $R_g$ respectively.

In general: $R = \frac{\mathcal{J}}{\mathcal{M}}$

\[ R_c = \frac{\mathcal{C}_c - \mathcal{G}}{\mathcal{M}_c} \quad R_g = \frac{\mathcal{G}}{\mathcal{M}_0} \]

\[ \mathcal{L} \text{ accounts for air gap not contributing to length, can also assume } \mathcal{L}_c - \mathcal{G} \approx \mathcal{L}_c \]

b. $L = \frac{N^2}{R_T}$

$R_T$ is the total reluctance. Since there is a series of reluctance here, it is the sum of $R_c$ and $R_g$.

\[ L = N^2 \left[ \frac{\mathcal{C}_c - \mathcal{G}}{\mathcal{M}_c} + \frac{\mathcal{G}}{\mathcal{M}_0} \right]^{-1} \]

\[ = N^2 \left[ \frac{\mathcal{M}_c \mathcal{M}_0 A}{(\mathcal{M}_c - \mathcal{M}_0 \mathcal{G}) + \mathcal{M}_0 l_c} \right] \]

This term missing if $\mathcal{L}_c - \mathcal{G} \approx \mathcal{L}_c$
6.1 \( L = N^2 \left( \frac{M_0}{M_0 + g + M_{le}} \right)^7 \)

It can be seen that \( L \) depends on \( A, g \), and \( M_{le} \). \( M_{le} \) has to be removed to make it insensitive to core permeability.

From before, the denominator was:
\[
\frac{lc - 5}{M_0 A} + \frac{9}{M_0 A}
\]

\[
\frac{lc - 5}{M_0 A}
\]

needs to be removed, so

\[
\frac{lc - 5}{M_0 A} \ll \frac{9}{M_0 A}
\]

\[
\Rightarrow \quad \frac{lc}{9} \ll 1 + \frac{M_0}{M_0}
\]

\[
\frac{lc}{9} \ll \frac{M_0}{M_0}
\]

for \( lc - 9 \approx lc \)

\( \beta = \gamma = N \phi \)

\( \phi \) is the flux \( BA \), so \( \beta_{sat} \) could be the max value.

\[
I_{\text{max}} = N \beta_{sat} A \leq \frac{\beta_{sat} \left[ (M_0 - M_0 g + M_{le}) \right]}{N M_0}
\]

\[
\leq \frac{\beta_{sat} (M_0 g + M_{le})}{N M_0}
\]

for \( lc - g \approx lc \).
6.2: The different reluctances will be characterized and drawn into a magnetic circuit in the same shape as the core to analyze.

\[ R_1 = \frac{e_1}{M_e A_1} \]
\[ R_2 = \frac{e_2}{M_e A_2} \]
\[ R_4 = \frac{e_4}{M_e A_2} \]

Notice how the center pillar wrapped by wire is included. The left and right paths have identical series reluctances, so the circuit can be simplified:

\[ R_1 = \frac{1}{2 (2R_2 + R_1)} \]

Due to the sum of series reluctances in parallel.

The total reluctance is described as

\[ R_1 = R_2 + \frac{1}{2 (2R_2 + R_1)} \]

\[ L = \frac{N^2}{R} = \frac{N^2}{\frac{e_1}{M_e A_1} + \frac{2e_2 + e_4}{2M_e A_2}} \]
Inductance \( L = \frac{N^2 \mu_0 A_c}{l_c} \)

\( A_c = h \cdot \frac{D_{out} - D_{in}}{2} = 16 \text{ mm}^2 \)

\( l_c = \pi \cdot \frac{D_{out} + D_{in}}{2} = 12\pi \text{ mm} \)

\( \mu_c = 25 \mu_0 = 25 \times 4\pi \times 10^{-7} \text{ H m}^{-1} \)

\( N = 10 \)

\[ L = 1.333 \mu H \]

\[ B = \frac{\phi}{A_c} = \frac{\mu_c N_i}{l_c} \]

\[ i_{\text{max}} = \frac{B_{\text{max}} l_c}{N \mu_c} = 60 \text{ A} \]
\[ R_1 = \frac{l_1}{\mu_0 A_1}, \quad R_2 = \frac{l_2}{\mu_0 A_2}, \quad R_3 = \frac{l_3}{\mu_0 A_3} \]

(Note: students are not expected to calculate these numbers)

\[ R_e = \frac{l_0}{\mu_0 A_e}, \quad \text{where} \quad l_e = 90.8 \, \text{mm}, \quad A_e = 107 \, \text{mm}^2 \]

provided in the datasheet.

b). \[ L = \frac{N^2 \mu_0 A_e}{l_e} = \frac{N^2}{R_e} \]

\[ N = 1 \implies L = 2813 \, \text{mH} = \frac{1}{R_e} \]

\[ A_L = 2813 \, \text{mH} \]

c). \[ B_{\text{max}} = \frac{\Phi}{A_{\text{min}}} = \frac{N_i}{R_e} \leq B_{\text{sat}} \]

\[ (N_i)_{\text{max}} = B_{\text{sat}} \times A_{\text{min}} \times R_e \]

\[ = B_{\text{sat}} \times A_{\text{min}} / A_L \]

\[ = 12.44 \, (\text{A. turns}) \]

d). for \( N = 12 \)

\[ i_{\text{max}} = 1.037 \, \text{A}. \]

Using \( A_1 \) as \( A_{\text{min}} \) in calculation. Students may use the \( A_{\text{min}} \) from datasheet. Also OK. 

\[ A_L = 2560 \, \text{mm}^2 \]

\[ (N_i)_{\text{max}} = 13.69 \, \text{A. turns} \]

\[ i_{\text{max}} = 1.1405 \, \text{A}. \]
e) Modified magnetic circuit:

\[ L_g = 250 \text{mm} \ll L_1 \]

\[ R'_1 = \frac{l_1 - L_g}{\mu_0 A_1} \approx \frac{L_1}{\mu_0 A_1} = R_1 \]

\[ R_g = \frac{L_g}{\mu_0 A_1}, \quad A_1 = \pi \left( \frac{11.3}{2} \right)^2 \]

\[ R_g = 1.984 \times 10^6 \, (H^{-1}) \]

\[ R_e = 1 / A_L = 3.55 \times 10^5 \, (H^{-1}) \]

\[ R'_e = R_g + R'_1 + R_2 + \frac{R_3}{x} \approx R_g + R_e \]

\[ L' = \frac{N^2}{R'_e} = \frac{N^2}{R_e + R_g} = 61.6 \, \text{mH} \]

\[ i_{\text{max}} = \frac{1}{N} B_{\text{sat}} \times A_{\text{min}} \times R'_e = 6.82 \, \text{A} \]

\[ E'_{\text{max}} = \frac{1}{2} L' i_{\text{max}}^2 = 1.433 \, \text{mJ} \]

\[ E_{\text{max}} = \frac{1}{2} L i_{\text{max}}^2 \]

\[ = \frac{1}{2} \times 12 \times A_L \times i_{\text{max}}^2 = 0.2178 \, \text{mJ} < E'_{\text{max}} \]

So the total energy storage capability for the gapped case is higher than the ungapped case.

(more than 6x higher)