5.1

\[ I_{\text{out}} = \frac{V_{\text{out}}}{R} \]

On the other hand, we can express \( I_{\text{out}} \) from the waveform above. These can give us an equation for \( V_{\text{out}} \), from which we can solve for it.

To express \( I_{\text{out}} \) from the waveform,

\[ I_{\text{out}} = \frac{1}{T} \int_{DT}^{t_1} i_L(t) \, dt \]

\[ = \frac{1}{T} \cdot \text{Area of shaded region}. \]

This is because \( i_{\text{out}} \) is supplied by \( i_L \) only during \( DT < t_1 \), when diode is on, switch is off. And \( \langle i_C \rangle = 0 \).

Area of shaded region

\[ = \frac{1}{2} i_{L,\text{pk}} \left( t_1 - DT \right) \]

\[ i_{L,\text{pk}} = \frac{V_{\text{in}}}{L} \cdot DT \]

\[ t_1 - DT = \frac{i_{L,\text{pk}}}{V_{\text{in}} - V_{\text{out}}} = \frac{V_{\text{in}} \cdot DT}{V_{\text{out}} - V_{\text{in}}} \]

Therefore, we get the equation for \( V_{\text{out}} \),

\[ \frac{1}{T} \cdot \frac{1}{2} \frac{V_{\text{in}} \cdot DT}{L} \cdot \frac{V_{\text{in}} \cdot DT}{V_{\text{out}} - V_{\text{in}}} = \frac{V_{\text{out}}}{R} \]

This is a quadratic equation, with two solutions.

Only the one greater than \( V_{\text{in}} \) is reasonable,

This gives us \( V_{\text{out}} = \frac{V_{\text{in}} + \sqrt{V_{\text{in}}^2 + 2V_{\text{in}}^2D^2TR}}{2} \)

So \( \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{2D^2RT}{L}} \)  

(For a different method, can refer to Book Krein P117-119)
5.2 (a) 

\[ i_{L,pk} = \frac{V_1 - V_2}{L} DT \]

\[ t_1 = DT + \frac{i_{L,pk}}{\frac{V_2}{L}} = \frac{V_1}{V_2} DT \]

(b) \[ \langle i_{ont} \rangle = \langle i_L \rangle = \frac{1}{T} \int_0^T i_L(t) \, dt \]

\[ = \frac{1}{T} \cdot \text{Area of shaded region} \]

\[ = \frac{1}{T} \cdot \frac{1}{2} \cdot i_{L,pk} \cdot t_1 \]

\[ = \frac{V_1(V_1 - V_2)DT}{2LV_2} \]
(c) At transition between CCM and DCM

The relationship \( V_2 = DV_1 \) still holds,

so \( I_{out} = \frac{V_2}{R} = \frac{DV_1}{R} \).

And we know from (b)

\[ I_{out} = \frac{V_1 (V_1 - V_2) D^2 T}{2L V_2}, \text{ with } V_2 = DV_1 \]

So

\[ \frac{V_1 (V_1 - DV_1) D^2 T}{2L \cdot DV_1} = \frac{DV_1}{R} \]

we get \( D = 0.7 \), which is the duty ratio corresponding to the transition between CCM and DCM.
a) Use KVL for the loop shown above,
\[ \langle V_i \rangle - \langle V_{L1} \rangle - \langle V_c \rangle + \langle V_{L2} \rangle = 0 \]
\[ \langle V_c \rangle = \langle V_i \rangle \]
\[ V_c = V_i \]

b) Use \[ \langle V_{L1} \rangle = 0 \]

0 ~ DT, M1 on, D off
\[ V_{L1} = V_i \]

DT ~ T, M1 off, D on
\[ V_i - V_{L1} - V_c - V_2 = 0 \]
\[ V_{L1} = -V_2 \]

\[ V_1 \cdot D + (-V_2)(1-D) = 0 \]
\[ \frac{V_2}{V_1} = \frac{D}{1-D} \]
The peak switch voltage occurs when the diode is on, and a KVL loop can be used.

\[ V_{sw} = V_{pk} = |V_i| + V_{o1} \]

The peak switch current occurs when the diode is off, so the switch has the full inductor current.

\[ I_{sw} = I_{pk} = I_L \]

To solve for \( I_L \), use the conservation of power and the fact that:

\[ |I_l| = DI_L \quad \text{and} \quad \frac{V_o}{V_i} = \frac{D}{1-D} \]

\[ I_L = \frac{1}{D} |I_l| = \frac{1}{D} \left( \frac{P_o}{V_i} \right) = \frac{1}{D} \frac{\frac{V_o}{V_i}}{V_i} = \frac{P_o}{1-D} \]

\[ \Rightarrow I_L = \frac{|I_l|}{D} = \frac{|I_o|}{1-D} \]

\( D \) can be solved for here in terms of \( I_l \) and then used to replace \( D \) in \( \frac{I_o}{i_l} \)

\[ D = \frac{|I_o|}{|I_l|} \]

\[ \Rightarrow \frac{V_o}{1-D} = I_L = \frac{|I_o|}{1-\frac{|I_l|}{|I_l|}} \]
\[ I_L = |I_i| = |I_o| \]

\[ I_L = |I_i| + |I_o| \]

\[ I_{sw} = I_{pk} = |I_i| + |I_o| \]

writing as \( \frac{I_i}{D} \) or other correct forms are fine.

The switch parameter can now be solved.

\[ V_{pk}I_{pk} = (V_i + 1V_0)(1I_i + 1I_o) \]

\[ = V_01I_i1 (\frac{V_i}{V_0} + 1) (\frac{I_i}{I_o} + 1) \]

\[ = P_0 \left( \frac{V_i}{V_0} + 1 \right) \left( \frac{|I_i|}{|I_o|} + 1 \right) \]

writing as \( P_0 \left( \frac{V_i}{V_0} + \frac{V_i}{V_0} + 2 \right) \) or other correct forms are fine.

b. Using KVL when a switch is off, their voltage stress can be determined since the diode on, their half of the circuit will be on.

\[ V_{M1, pk} = |V_i| \]

\[ V_{M2, pk} = |V_o| \]
Let the current through the inductors be $I_m$.

Through the usage of kV, it can be seen that the peak current through both switches is $I_m$.

$\begin{align*}
I_{m1, pk} &= \left| I_m \right| \quad I_{m2, pk} = \left| I_m \right|
\end{align*}$

Through conservation of power, all output power must pass through $V_m$.

$\Rightarrow P_o = V_m I_m$

The switch stress can be solved for using conservation of power and substitution.

$\begin{align*}
V_{m1, pk} I_{m1, pk} &= |V_i| |I_m| \\
V_{m2, pk} I_{m2, pk} &= |V_o| |I_m|
\end{align*}$

$\begin{align*}
= |V_i| \frac{P_o}{|V_m|} \\
= |V_o| \frac{P_o}{|V_m|}
\end{align*}$

$\begin{align*}
V_{m1, pk} I_{m1, pk} = P_o \frac{|V_i|}{|V_m|} \\
V_{m2, pk} I_{m2, pk} = P_o \frac{|V_o|}{|V_m|}
\end{align*}$

1. Two cases: Boost or buck the input only.

$\Rightarrow \quad 50V \leq V_i \leq 100V$ or $100V \leq V_i \leq 200V$ respectively due to one converter necessary in each range.

2. $50V \leq V_i \leq 100V$ Boost only.

Since boost is utilized, ignore the buck and see $V_m$ as input. This means $D_o = 1$

$\frac{|V_o|}{|V_i|} = \frac{1}{1 - D_o} \Rightarrow \frac{|V_o|}{|V_i|} = \frac{1}{1 - 1} = \frac{1}{0}$

Stress is inversely related to $|V_m|$, so $|V_m|$

has to be maximized to $|V_i|$

2. $100V \leq V_i \leq 200V$ Buck only.
To ignore the boost, $D_u = 0$. The conversion ratio is now the same as a regular buck.

\[
D_o = \frac{V_{il}}{V_{il} + D_o} \quad \text{Treat } \frac{V_{il}}{V_{il} + D_o} \text{ because the boost now results in no change.}
\]

\[
\frac{V_{ol}}{V_{il}} = 1 \quad D_o = 1
\]

To minimize stress, $\frac{V_{il}}{V_{ol}}$ needs to be maximized to $V_{ol}$.

The total stress can be found by summing the answers from 5.4b and substituting for both cases.

1. $50 \text{ V} \leq V_{il} \leq 1000 \text{ V}$

Total stress = $P_o \left( \frac{V_{il}}{V_{il}} + \frac{V_{ol}}{V_{il}} \right) = P_o \left( 1 + \frac{V_{ol}}{V_{il}} \right)$

\[
\text{Let } \frac{V_{il}}{V_{il}} = 1
\]

\[
D_o = 1
\]

\[
D_u = 1 - \frac{V_{il}}{V_{ol}}
\]

2. $100 \text{ V} \leq V_{il} \leq 200 \text{ V}$

Total stress = $P_o \left( \frac{V_{il}}{V_{il}} + \frac{V_{ol}}{V_{il}} \right) = P_o \left( \frac{V_{il}}{V_{il}} + 1 \right)$

\[
\text{Let } \frac{V_{il}}{V_{il}} = 1
\]

\[
D_o = \frac{V_{ol}}{V_{il}}
D_u = 0
\]
\[ V_i: \text{From } a: \frac{P_o}{(\frac{V_i}{V_{ol}} + 1)(\frac{V_{ol}}{V_i} + 1)} \]
\[ \text{From } c: \frac{P_o}{(\frac{V_i}{V_{ol}} + 1) \text{ or } P_o (1 + \frac{V_{ol}}{V_i})} \]

In both cases, because each case essentially drops \( \frac{V_i}{V_{ol}} \) or \( \frac{V_{ol}}{V_i} \) from \( a \), resulting in \((1 + 1)\) and a lower value.

**Circuit b.**

d. **Size:** b is potentially larger

- **Components:** b requires more parts
- **Control:** b has two independent switching signals
- **Output:** a inverts the output