University of Illinois at Urbana Champaign
Department of Electrical and Computer Engineering

ECE 464 – Power Electronics
Problem Set 10
Reading: Section

Motivation: This problem set is designed to expose you to the analysis of dc-ac converters.
Reading: 4.5.1, 5.3-5.4, 6.5.3

Problem 10.1
During a visit to see her family over Thanksgiving break, ECE 464 student Adrianna Turner discovers that her father has purchased a solar PV installation. Her father proudly boasts that he saved a significant amount of money by buying the inverter from eBay, and installing it himself by simply plugging the inverter output into his grid outlet. Since Adrianna has learned about the ill-effects of harmonics and specific regulations required for grid-connected inverters (IEEE Standard 1547), she is curious to see what the THD is of the current that this low-cost solar inverter outputs. Since she does not have the best measurement equipment at his parents' house, all she can record is the solar inverter output current ripple shape (whose fundamental is in phase with the voltage), as shown in Figure 1. Please help Adrianna calculate the THD of this waveform. Also, please comment on whether it meets IEEE 1547 requirements with respect to THD. [Hint: You should look for IEEE 1547 and check lecture notes]

Problem 10.2
Figure 10.2(a) shows a full bridge voltage source converter (a.k.a. H-Bridge inverter) used to create ac waveforms at the output (denoted as $V_{\text{bridge}}$) from a dc voltage source (denoted as $V_{\text{DC}}$). For part (a)-(d), the converter switches are controlled to create a $V_{\text{bridge}}$ waveform as shown in Fig. 10.2(b). For part (e) and (f), the same converter is controlled to create a $V_{\text{bridge}}$ waveform as shown in Fig. 10.2(c).

a) Find an expression for the THD of the waveform $V_{\text{bridge}}$ in Fig. 10.2(b) as a function of $\delta$.
b) Find the Fourier series expression of the voltage $V_{\text{load}}$, shown in Fig. 10.2(a), given resistance $R$, capacitance $C$, inductance $L$, and switching frequency $f_{\text{sw}}$. [Hint: In this case, the switching frequency is same as the fundamental frequency of the output waveform]
c) Find an expression for the THD of the voltage $V_{\text{load}}$ as a function of $R$, $C$, $L$, and $f_{\text{sw}}$.
d) Using MATLAB, plot the expression in (a) and (c) for $\delta$ over a range 0-80 degrees. For this question, you may assume the following operating conditions: $f_{\text{sw}} = 60$ Hz, $R = 100$ $\Omega$, $C = 8.94$ $\mu$F, $L = 160.6$ mH.
e) For the bridge voltage waveform, $V_{\text{bridge}}$ of Fig. 10.2(c), find the amplitudes of the fundamental, third,
and fifth harmonic. [Hint: you may want to sketch $\sin(3\omega t)$ and $\sin(5\omega t)$ on top of the bridge voltage waveform, and use symmetry to minimize the amount of math required].

f) Find the total harmonic distortion (THD) of the waveform $V_{\text{bridge}}$ of Fig. 10.2(c).

Figure 10.2: An H-Bridge inverter with different switching pattern

Problem 10.3 [Multilevel Inverter with flying capacitor]
Figure 10.3 shows the schematic of a full-bridge single-phase three-level inverter. Similarly to the one-leg three-level inverter discussed in lecture, the flying capacitors $C_A$ and $C_B$ are controlled to each have a voltage that is approximately half of the input voltage $V_{DC}$ (by balancing the switching patterns

Figure 10.3: Multilevel Inverter with a Flying Capacitor
appropriately).

a) Propose a switching pattern for the devices in the flying capacitor inverter that takes advantage of the multilevel capabilities of the inverter and results in an (unfiltered) output voltage $V_{L}(t)$ having no 3rd, 5th, or even harmonics.

b) You may assume that the voltages on the flying capacitors are at one half the input voltage. Note that each switch is switched oppositely with its corresponding “primed” switch. Consequently, the independent switching functions are $q_{A1}$, $q_{A2}$, $q_{B1}$, and $q_{B2}$. How many times does each active device switch on and off per ac output cycle?

c) How does the total harmonic distortion of the unfiltered waveform $V_{L}(t)$ compare to that of a conventional two-level inverter of Problem 10.2?

**Problem 10.4**

Shown in Figure 10.4 is a schematic drawing of a 2 kW inverter that converts 400 V DC to 240 Vrms AC. Also shown are instantaneous voltage and current waveforms delivered to the AC grid, along with instantaneous power. To temporarily store the instantaneous difference between power delivered to the grid, and drawn from the source (modeled as DC current source here), capacitor $C_b$ is used.

a) Calculate the amount of energy that must be stored and released in capacitor $C_b$ in each twice-line-frequency period.

b) If the voltage on the DC bus is allowed to ripple by $\pm$ 5 V (i.e., from 395 to 405 V), how much capacitance must be used to provide this amount of energy storage?

![Figure 10.4: A dc to ac converter](image)

**Problem 10.5** [I did not break my promise of asking LTSpice Simulation; we are providing you the basic LTSpice model, all you need to do is write the switching functions]

Please download the LTSpice Model of an H-Bridge dc-ac converter. A screenshot of the model is shown in Fig. 10.5(a). The model is set to operate with an input dc voltage of 100 V, and create an ac at the output driving an R-L load with $R = 8 \ \Omega$ and $L = 1 \ mH$. The switching frequency of the converter is 5 kHz (as set by the triangle carrier wave frequency), while the fundamental frequency of the ac source is 200 Hz (as set by the modulating sine wave function). Running the model and by probing the behavioral
voltage source, B6 marked as AC Output Voltage, you will find out the output AC waveform. The Fourier spectrums of the output voltage waveform (\text{AC\_Output}) and the current waveform (\text{I(Resistor)}) are shown in Fig. 10.5 (b) and 10.5(c), respectively.

![LT Spice Simulation](image)

(a)

![Fourier Spectrum of Output Voltage](image)

(b)

![Fourier Spectrum of Current](image)

(c)

Figure 10.5: LT Spice Simulation of different PWM switching strategies that we studied in class.

a) Can you identify the switching frequency from the Fourier Spectrum? Write down the rms magnitude and the corresponding frequency of the most dominant component of the output voltage waveform apart from the fundamental component, which is supposed to be at 200 Hz. Be
mindful, the frequency is plotted in a log-scale.

b) Which switching strategy is being used in this simulation? Strategy I, II, or III, as per the class notes (see resources section in Piazza)?

c) Simulate the other two switching strategies. Print and submit the Fourier Spectrum of the output voltage waveform and the current waveform. Write down the rms magnitude and the corresponding frequency of the most dominant component of the output voltage waveform apart from the fundamental component for each of these two strategies. [Hint: You need to come up with mathematical functions for the behavioral voltage sources that are acting as comparators. Allowable math functions are: 
http://ltwiki.org/LTspiceHelp/LTspiceHelp/B_Arbitrary_behavioral_voltage_or_current_sources.htm Also, you may need additional sine functions, behavioral voltage sources etc.]

Happy Thanksgiving!!