



Power Electronics Day 9 -- Magnetics

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Introduction to Magnetism

- First, notation:

$$\oint_S \epsilon \mathbf{E} \cdot d\mathbf{s} = \int_V \rho_v \, dv \quad \text{Gauss' Law}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0 \quad \text{Gauss' Law (magnetic fields)}$$

$$\oint_\ell \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{s} \quad \text{Faraday's Law}$$

$$\oint_\ell \mathbf{H} \cdot d\boldsymbol{\ell} = \int_S \mathbf{J} \cdot d\mathbf{s} + \frac{\partial}{\partial t} \int_S \epsilon \mathbf{E} \cdot d\mathbf{s} \quad \text{Ampere's Law}$$

$$\oint_S \mathbf{J} \cdot d\mathbf{s} = -\frac{d}{dt} \int_V \rho_v \, dv \quad \text{Conservation of charge}$$





Magnetic Field System

- In almost any converter, the electric fields are not large.
- The currents are likely to be considerable.
- This supports simplification.
- The displacement current term in Ampere's Law can be ignored.



Magnetic Field System

- We also define flux ϕ .

$$\int_s \mathbf{B} \cdot d\mathbf{s} = \phi \quad \text{Definition of flux}$$

$$\oint_s \mathbf{B} \cdot d\mathbf{s} = 0 \quad \text{Gauss' Law (magnetic fields)}$$

$$\oint_l \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_s \mathbf{B} \cdot d\mathbf{s} \quad \text{Faraday's Law}$$

$$\oint_l \mathbf{H} \cdot d\mathbf{l} = \int_s \mathbf{J} \cdot d\mathbf{s} \quad \text{Ampere's Law}$$



Materials

- Permeability μ .
- Vacuum has $\mu_0 = 4\pi \times 10^{-7}$ H/m.
- Materials:
 - Diamagnetic $\rightarrow \mu < \mu_0$
 - Paramagnetic $\rightarrow \mu > \mu_0$
 - Ferromagnetic $\rightarrow \mu \gg \mu_0$
 - Superconductors $\rightarrow \mu \sim 0$
- Diamagnetic and paramagnetic materials are still close to μ_0 .





Materials

- We are interested in ferromagnetic materials for applications in devices, since they are much different from air or conductors.
- Superconductors are also of interest (future converters).





Materials

- Ferromagnetic elements
 - Iron, nickel, cobalt
 - Some rare earths: gadolinium, dysprosium (but low Curie temperature)
- Compounds
 - Chromium oxide
 - Oxides of iron, nickel, ...
- Alloys
 - Manganese, aluminum, zinc, rare earths, ...





Materials

- Most common for us:
 - Magnetic steels (good for low frequency)
 - Ferrites (magnetic oxides mixed and assembled in ceramic form)
 - Pure powdered iron in a ceramic matrix.
 - Other powdered alloys.
- Others of interest:
 - Permanent magnets: samarium cobalt, neodymium-iron-boron, ferrites
 - High-temperature superconductors



Magnetic Circuits

- Faraday's Law is KVL if we *define* a changing flux $d\phi/dt$ as voltage (EMF).
- Ampere's Law looks similar, if the term on the right side is defined as MMF.

$$\oint_l \mathbf{H} \cdot d\mathbf{l} = \int_s \mathbf{J} \cdot d\mathbf{s} .$$



Magnetic Circuits

- KVL:

$$\sum_{loop} EMF = 0$$

- Ampere:

$$\sum_{loop} MMF = 0$$





Magnetic Circuits

- Ampere's Law becomes like a "KVL for MMF."
- Gauss' Law (magnetic fields) becomes like "KCL for flux."
- We take MMF as a forcing variable and flux as a flow variable.





Magnetic Circuits

- Gauss' Law

$$\oint_S B \cdot ds = 0$$

- Define a “flux node.”

$$\sum_{\text{closed region}} \phi = 0$$

- We have:

$$\sum_{\text{node}} \phi = 0$$

- This is like “KCL for flux.”





Magnetic Circuits

- Ampere's Law acts like "KVL for MMF."
- Gauss' Law (magnetic fields) acts like "KCL for flux."
- This supports a "magnetic circuit" simplification of the equations.
- First, the equations directly.



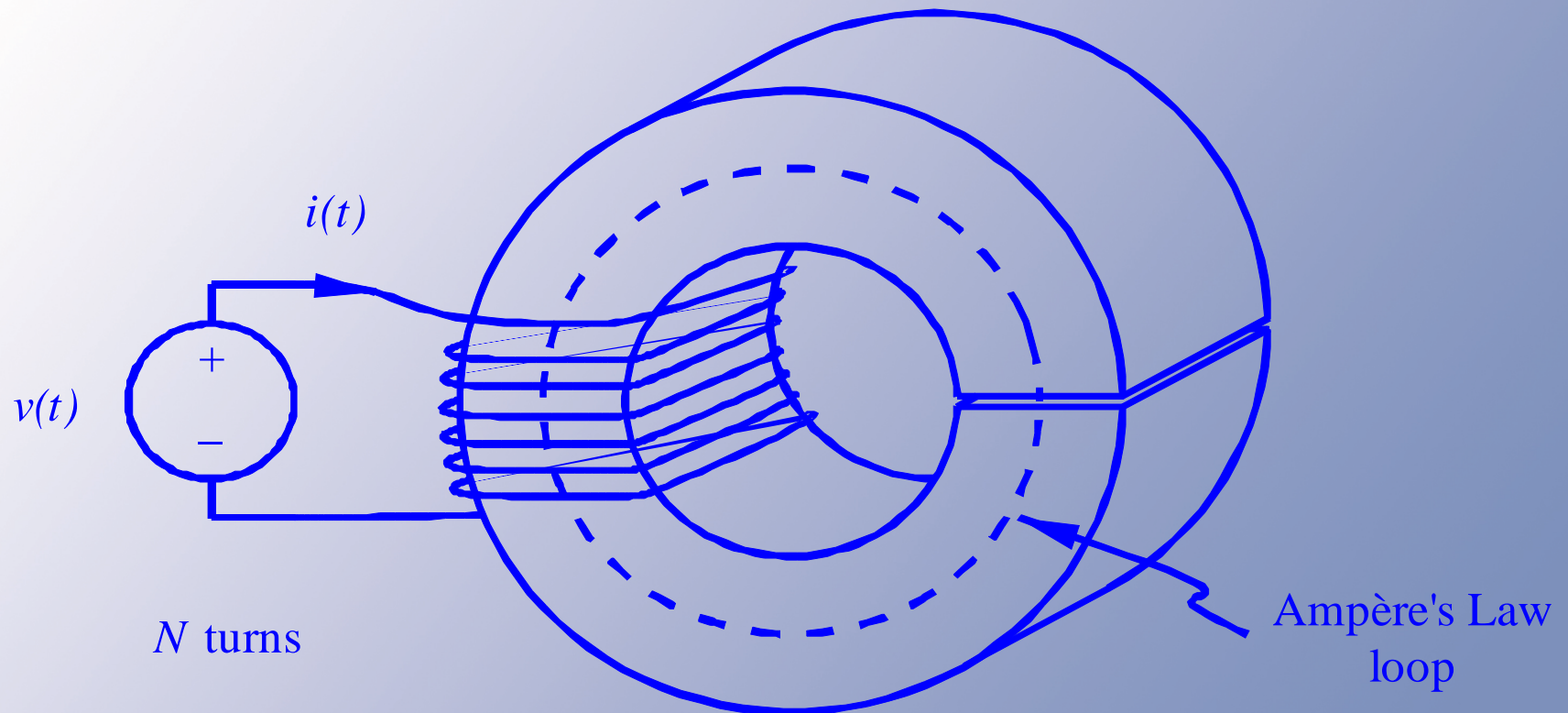


Inductance

- A coil of wire around a ferromagnetic core.
- **Faraday's Law:** Voltage applied to the coil should produce a rate of change of flux.
- **Ampere's Law:** Current produces MMF.



Coil on a Core



- Faraday's Law: loop is the wire.
- Ampere's Law: loop is in the core.



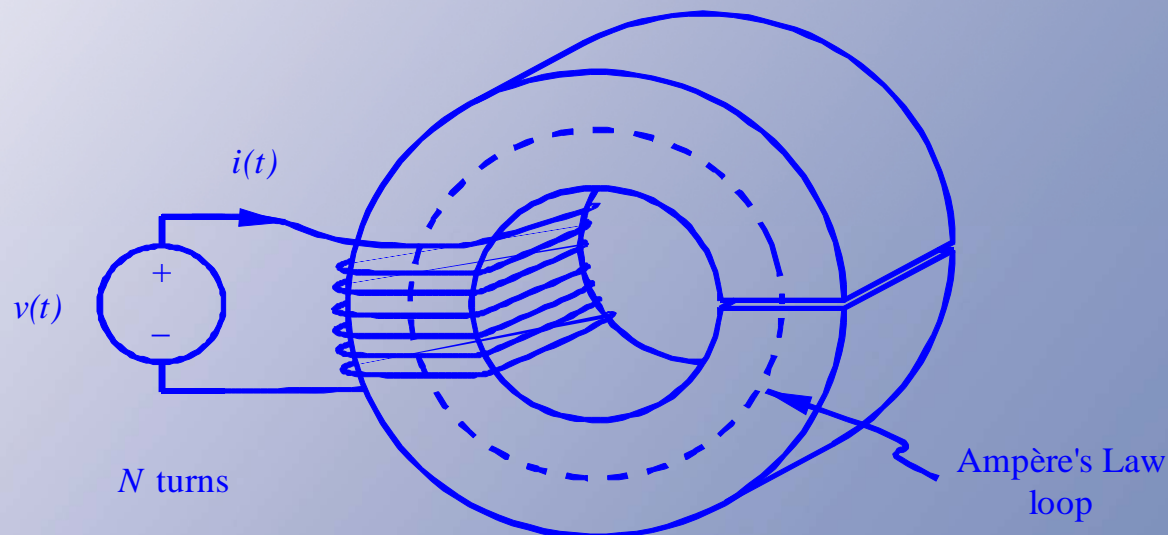
Inductance

- Write Faraday's Law and Ampere's Law for this arrangement.
- We define flux linkage, λ , as $N\phi$.
- Faraday's Law can be written in a tighter notation: $v_{in} = d\lambda/dt$.



Ampere's Law

- For Ampere's Law, we can draw a loop through the center of the core.
- Assuming (arbitrarily for now) that H is the same throughout the core, the integrals become $H_{\text{core}}\ell_{\text{core}} + H_{\text{air}}\ell_{\text{air}} = Ni$, and $\mu H = B$.



Ampere's Law

- Re-write in terms of ϕ (we're ultimately trying to link Ampere's Law and Faraday's Law).

$$B_{\text{core}} \ell_{\text{core}} / \mu_{\text{core}} + B_{\text{air}} \ell_{\text{air}} / \mu_{\text{air}} = Ni.$$

- But $\phi = B A$, with A as the cross-section area.
- $\phi_{\text{core}} \ell_{\text{core}} / (\mu_{\text{core}} A_{\text{core}}) + \phi_{\text{air}} \ell_{\text{air}} / (\mu_{\text{air}} A_{\text{air}}) = Ni.$



Reluctance

- We define reluctance, \mathcal{R} , as $\ell/(\mu A)$ for a material.
- This means $\phi_{\text{core}}\mathcal{R}_{\text{core}} + \phi_{\text{air}}\mathcal{R}_{\text{air}} = Ni$.
- Now, the magnetic circuit idea: Since ϕ is flow and $\text{MMF} = Ni$ is forcing, the $\phi\mathcal{R}$ terms are an “MMF drop,” and Ni is an “MMF source.”



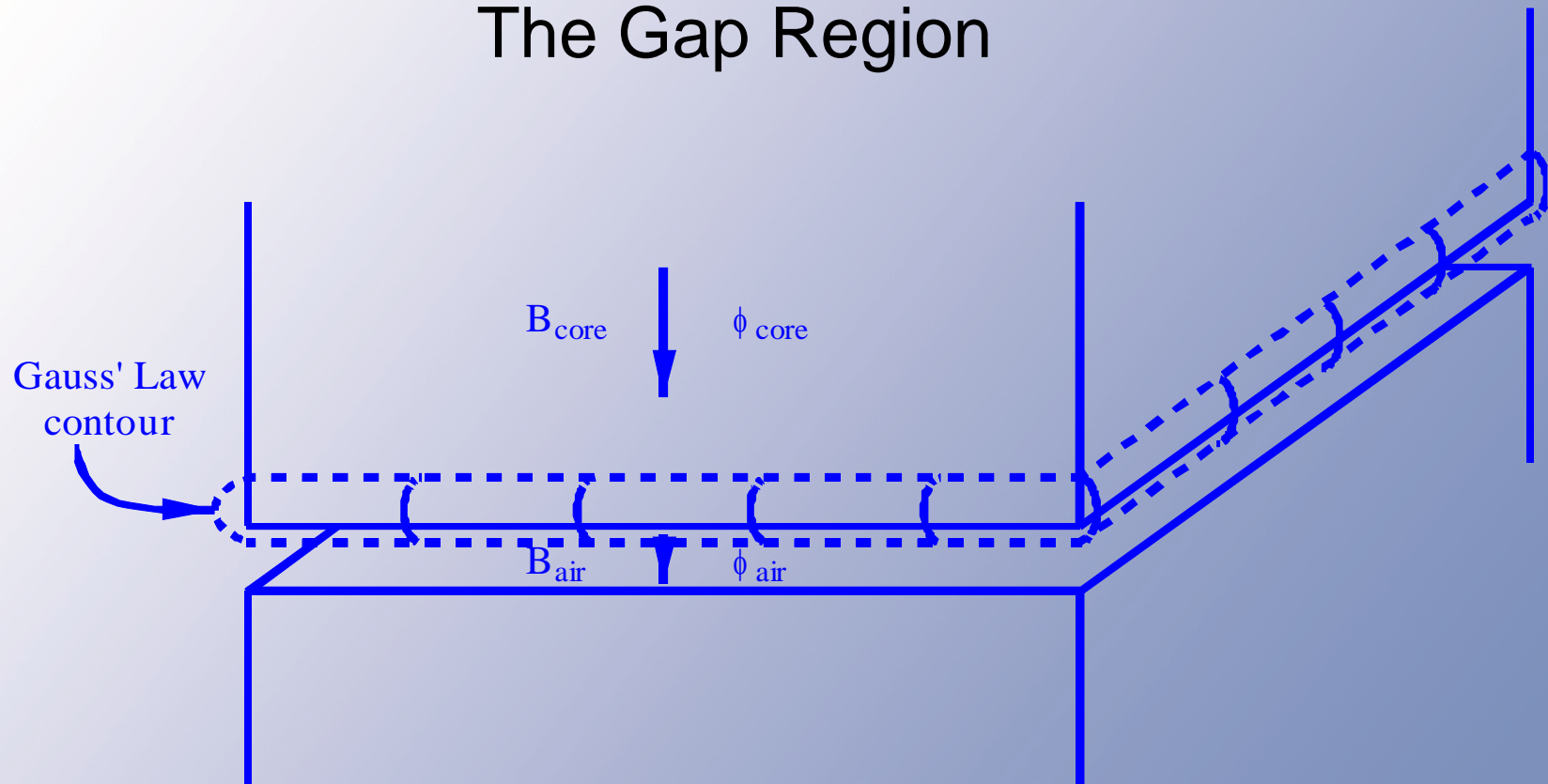


Flux Relations

- But two fluxes appear. Can we relate them?
- Define a “node:” the end of the core as it interfaces with the air gap.
- This region meets Gauss’ Law.



The Gap Region



- Gauss' Law applies to the dotted volume.



Flux Relations

- If a very thin region is used, the total flux entering its surface is $B_{\text{core}}A_{\text{core}} - B_{\text{air}}A_{\text{air}} = 0$.
- These are the fluxes $\phi_{\text{core}} - \phi_{\text{air}} = 0$.
- Thus Gauss' Law means there is only one flux, ϕ .
- Now $\phi(\mathcal{R}_{\text{core}} + \mathcal{R}_{\text{air}}) = Ni$, or $\phi\mathcal{R}_{\text{tot}} = Ni$.





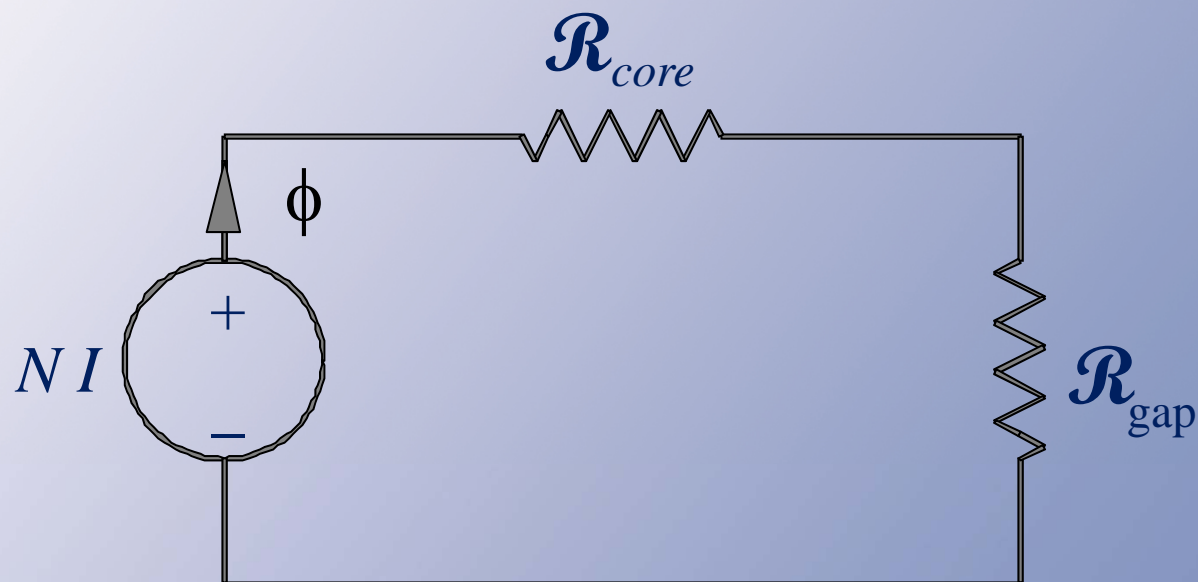
Back to Faraday's Law

- Take time derivatives. Reluctance is constant.
 $\mathcal{R}_{\text{tot}} d\phi/dt = N di/dt$, and $\lambda = N\phi$.
- Rewrite: $d\lambda/dt = N^2/\mathcal{R}_{\text{tot}} di/dt$.
- The coil voltage is related to current as $v_{\text{in}} = N^2/\mathcal{R} di/dt$.
- N^2/\mathcal{R} defines an inductance, L .
- Manufacturers provide a per-unit inductance, A_L equal to $1/\mathcal{R}$. Typical values: nanohenries per turn².



Magnetic Circuit

- The equations we built can be represented with a circuit:





Magnetic Circuits

- Magnetic structures can be constructed as reluctance circuits of this type.
- This supports identification of inductance and analysis of devices.
- It does not model any losses.





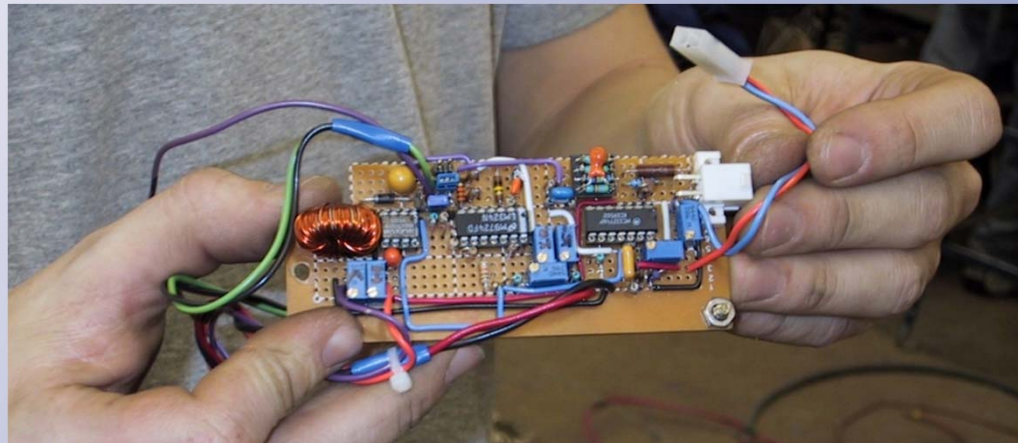
Reluctance

- The magnetic circuit analogue relations are given in Table 12.2.
- A “magnetic conductor” has high permeability and low reluctance.
- A “magnetic insulator” has high reluctance.



Limitations

- In electric circuits, the conductors are very good, and the insulators are nearly perfect.
- The ratio of resistance in a circuit to resistance of the insulation can be lower than 10^{-20} .



- This is not true in magnetics.

Limitations

- The reluctance ratio between a core and the surroundings can be more than 10^{-3} , and is rarely smaller than 10^{-5} .
- This is like taking the insulation off a circuit and operating it in a pail of salt water.





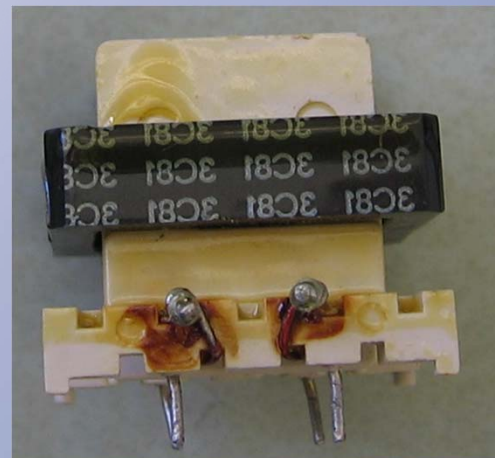
Leakage

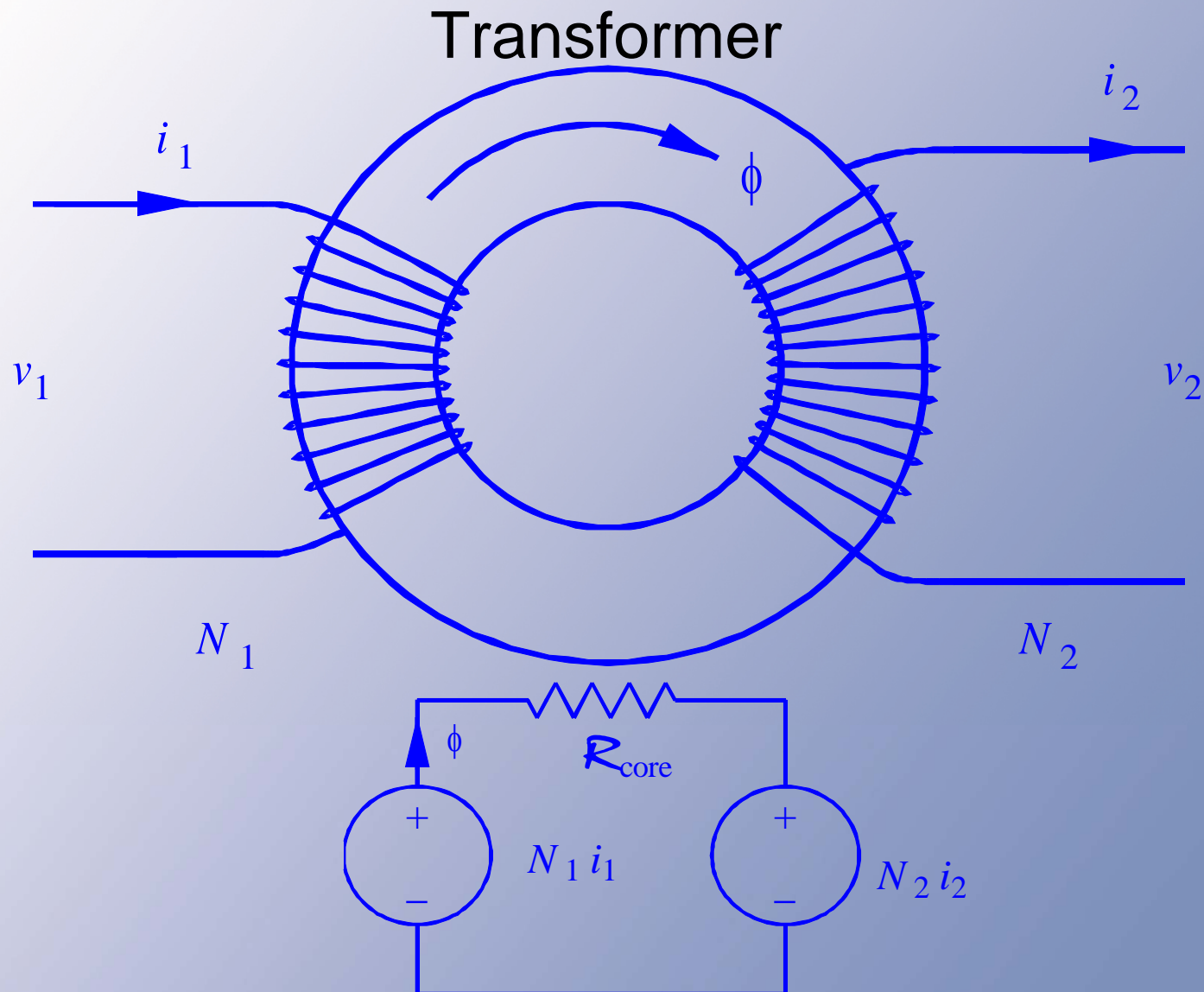
- This analogy suggests that much of the flux leaks out into the surroundings.
- Leakage is an important issue.
- Also, the value μ_{core} comes from a nonlinear relation, and is not constant.

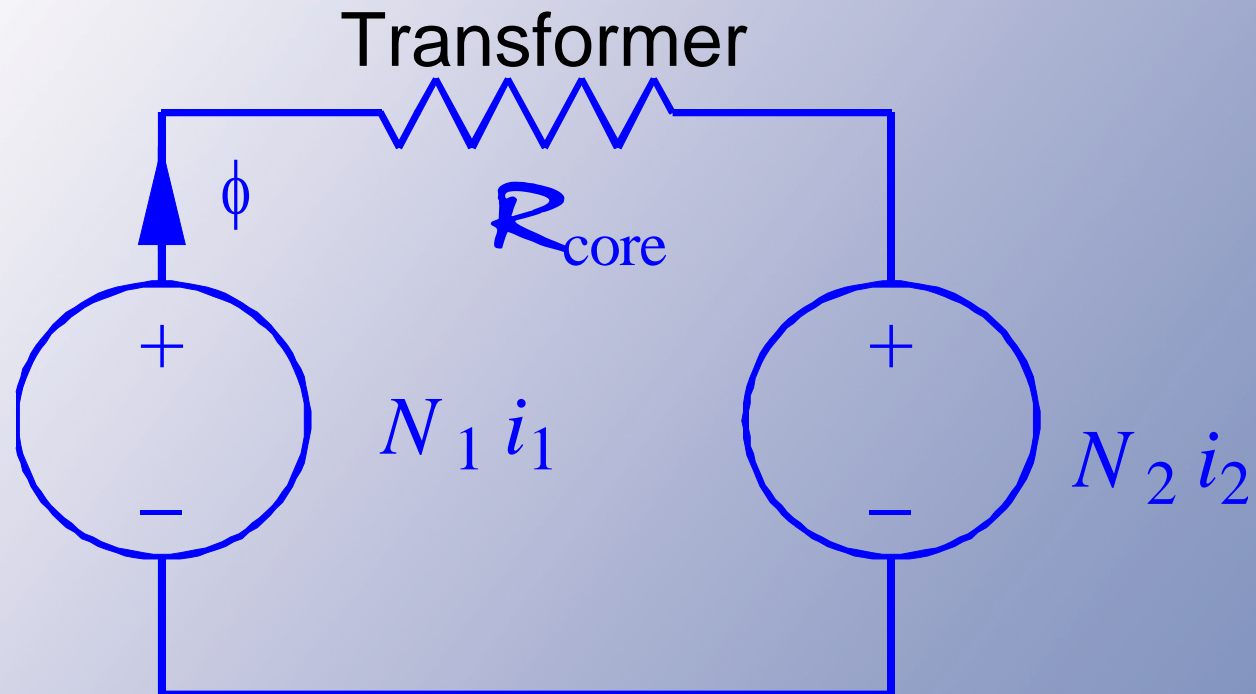


Transformers

- A transformer has two windings, and the net MMF balances the reluctance MMF drop.
- If core reluctance is very low, there is an MMF balance, $N_1 i_1 = N_2 i_2$. With flux ϕ , the voltages are $v_1 = N_1 d\phi/dt$, $v_2 = N_2 d\phi/dt$.







- Ideal case requires $N_1 i_1 = N_2 i_2$.
- $v_1 = d\lambda_1/dt = N_1 d\phi/dt$
- $v_2 = d\lambda_2/dt = N_2 d\phi/dt$

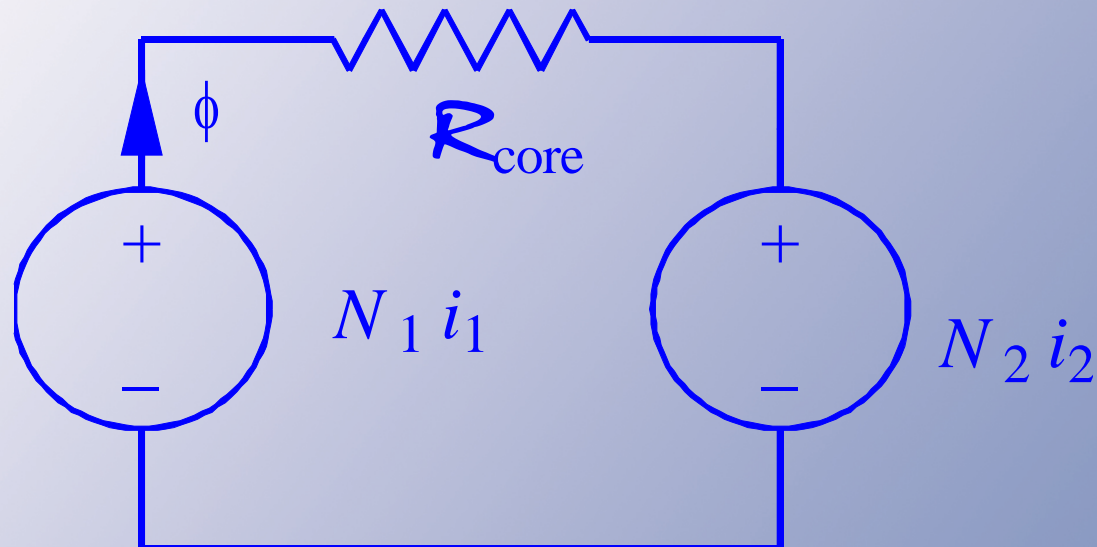


Ideal Case

- If the derivative $d\phi/dt$ is nonzero, the voltage ratio $v_1/v_2 = N_1/N_2$.
- A very low reluctance core yields an ideal (but ac) transformer.
- The reality is that reluctance is nonzero, and flux flows even if i_2 is set to zero.



Transformer



- Real case: $N_1 i_1 \neq N_2 i_2$.
- There is an “MMF drop,” associated with the magnetizing inductance.



Ideal Coupled L

- The nonzero reluctance yields inductance $L_1 = N_1^2/\mathcal{R}$, seen from the #1 side, and $L_2 = N_2^2/\mathcal{R}$, seen from the #2 side.
- This supports the coupled inductor application for flyback converters.





Ideal Cases

- An ideal transformer has low reluctance. No stored energy.
- An ideal coupled inductor has significant reluctance, but no other effects. This is the *magnetizing inductance*, and it can store energy.



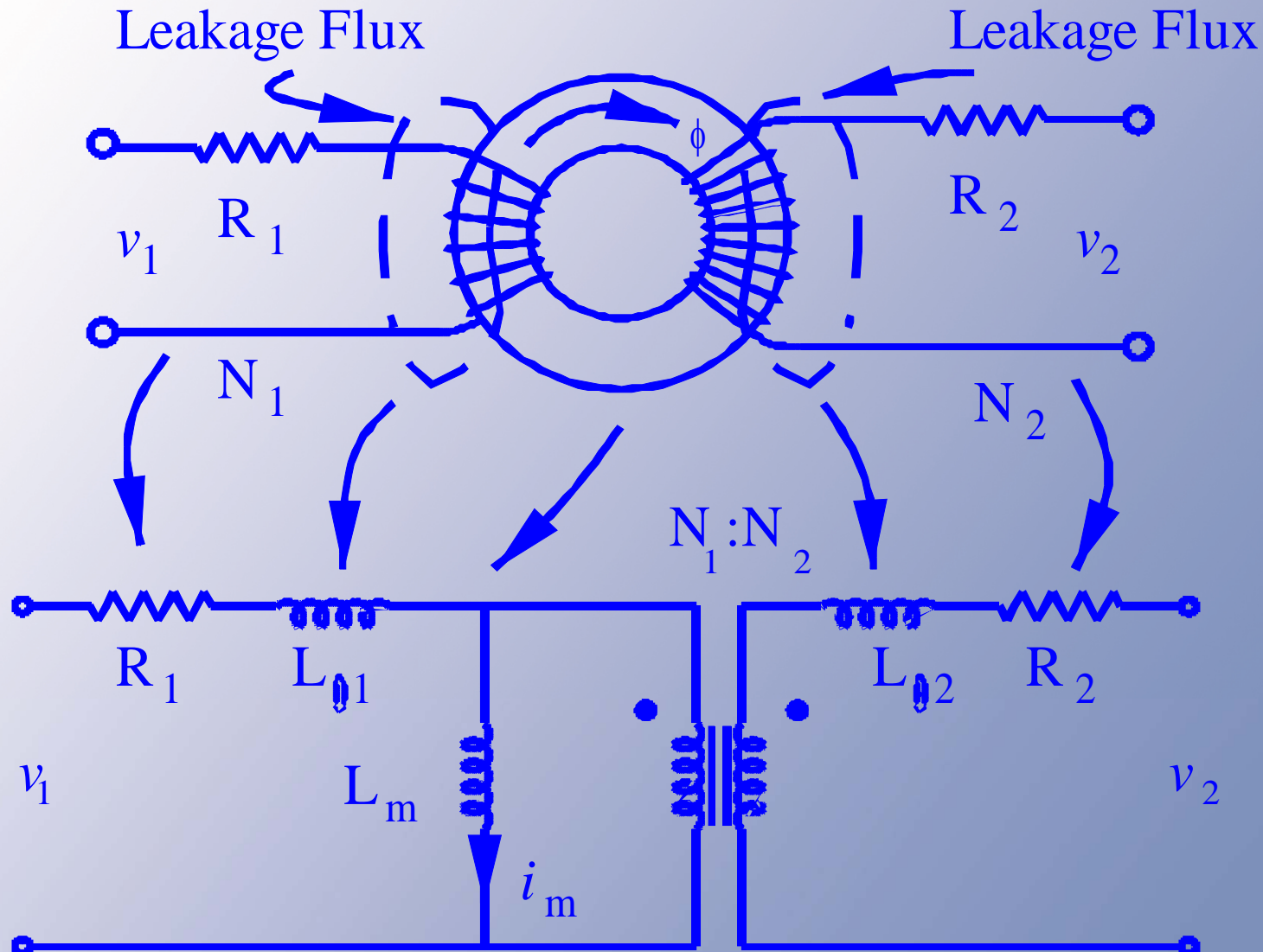


Real Cases

- The cores must be wound with wire -- series resistance is introduced.
- There is leakage flux, and such flux flows through its own reluctance path, forming an inductance.



Device Circuit Model





Some Points

- The resistances are rarely negligible, since limited space means limited copper.
- The leakage can be low, but it still introduces “stray” stored energy.
- This does not model losses in the core itself.





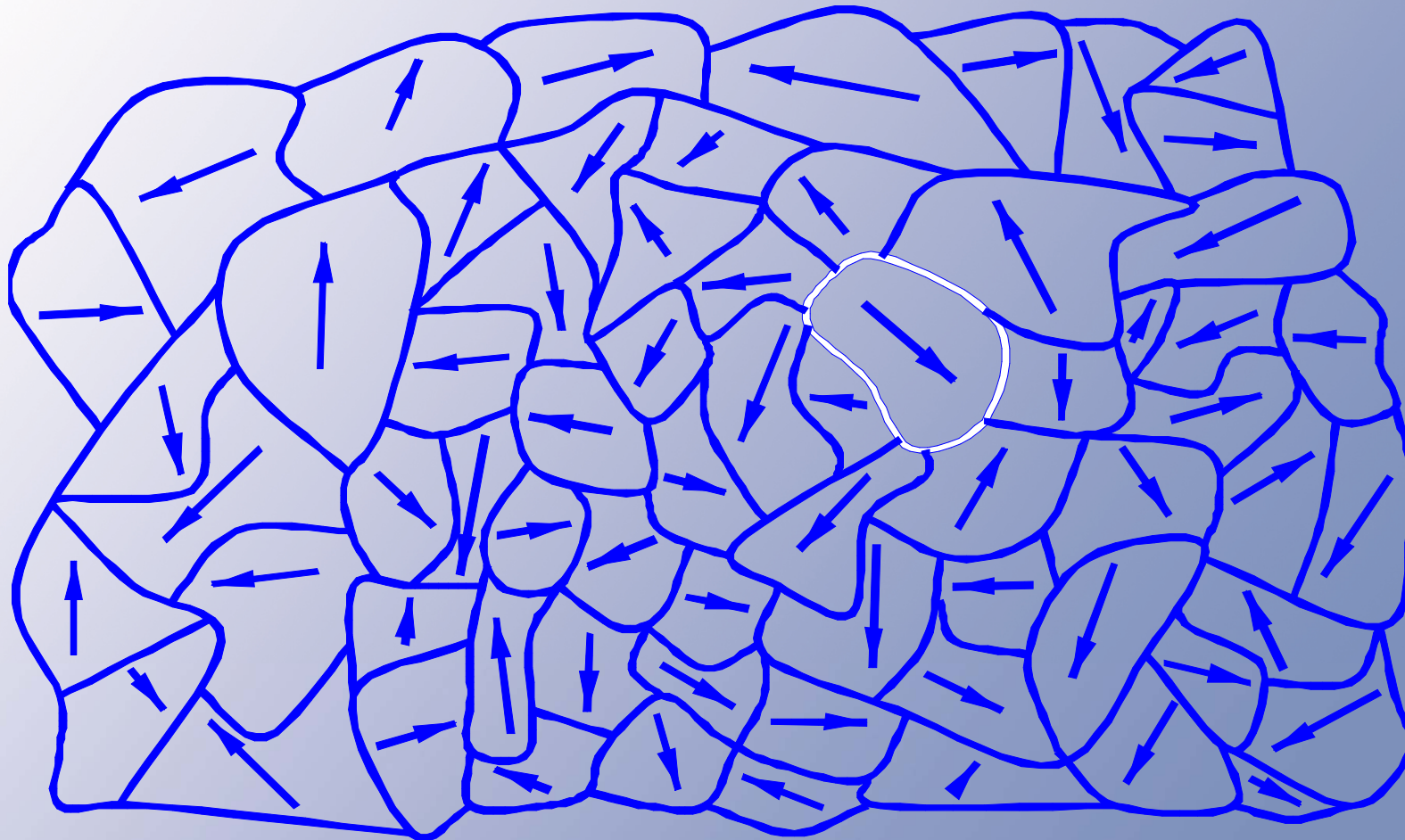
Hysteresis

- For ferromagnetic materials, the basis of high permeability is inherently nonlinear.
- The process is not entirely reversible.
- This hysteresis effect gives rise to losses.





Domain Alignment





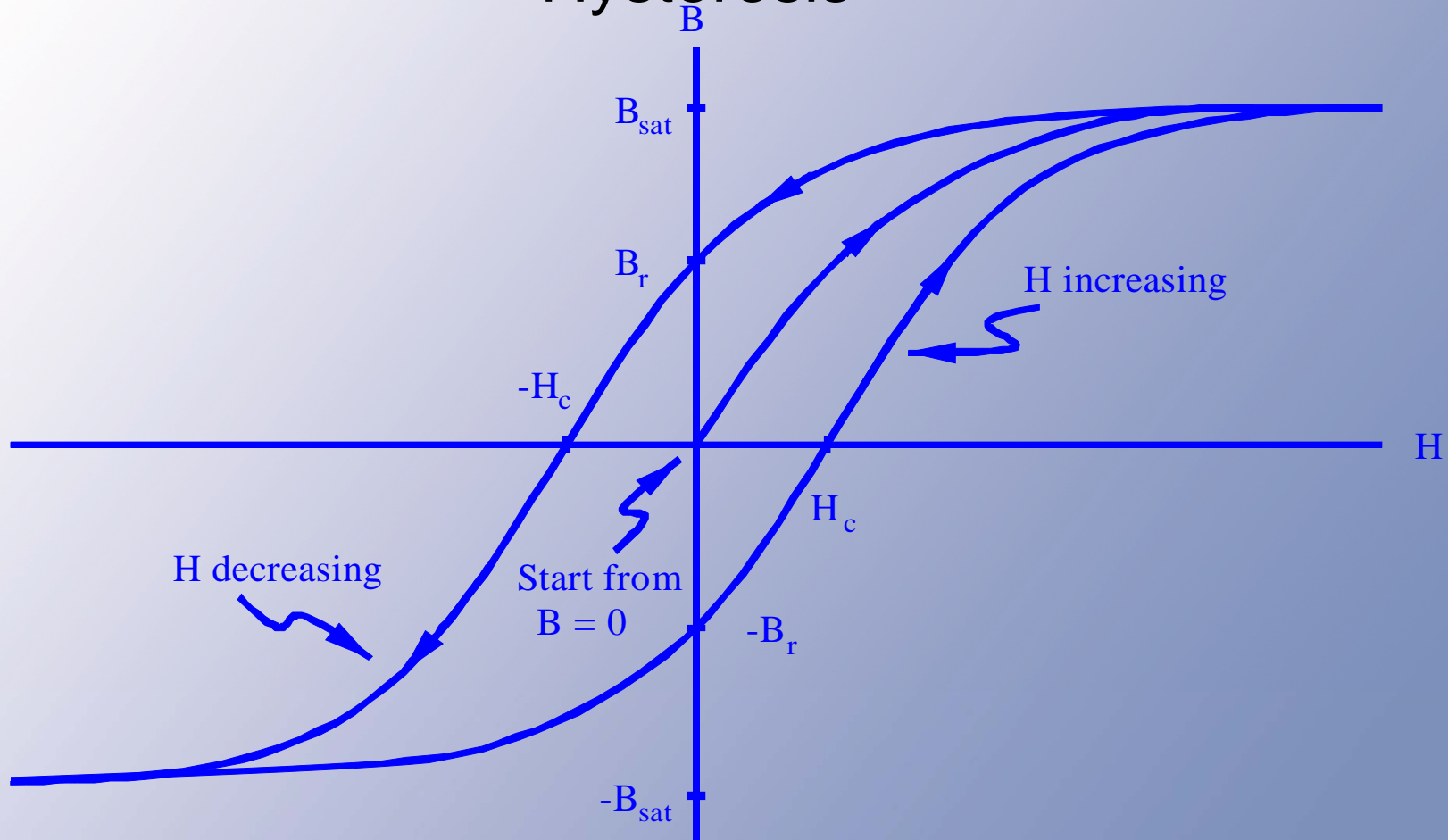
Hysteresis

- Since domains are a large group of atoms that align together, small MMF changes yield large flux changes.
- This implies high μ values.
- But it is nonlinear: once domains are aligned, they cannot add much more flux.





Hysteresis





Hysteresis

- The units within the loop ($B \times H$) are energy per unit volume.
- A certain energy (per unit volume) is lost each time the loop is traversed.
- At a frequency f , there is a power loss of f times the loss represented by the loop.





Hysteresis

- With hysteresis, we see that a real magnetic device has loss within the core.
- In addition, the core is a little bit conductive, which leads to internal i_2R loss.





Basis of Losses

- There are losses in windings (“copper losses”) and losses in the magnetic core.
- Domain alignment irreversibility: gives hysteresis loss each time around the loop. This is approximately proportional to frequency.
- Eddy current losses: The field induces current within the core.



Basis of Losses

- Steel cores are built with thin, insulated, layers to reduce conductive path lengths.
- These lamination methods work adequately up to a few kHz.
- For higher frequencies, poorly conducting ceramics (ferrites) are a better solution.
- Ferrites can function to several MHz.





Basis of Losses

- Eddy current loss depends both on field strength and on frequency.
- Since $v = NA \, dB/dt$, and loss depends on v^2 , we expect losses proportional to B^2 and to f^2 .





Core Loss Summary

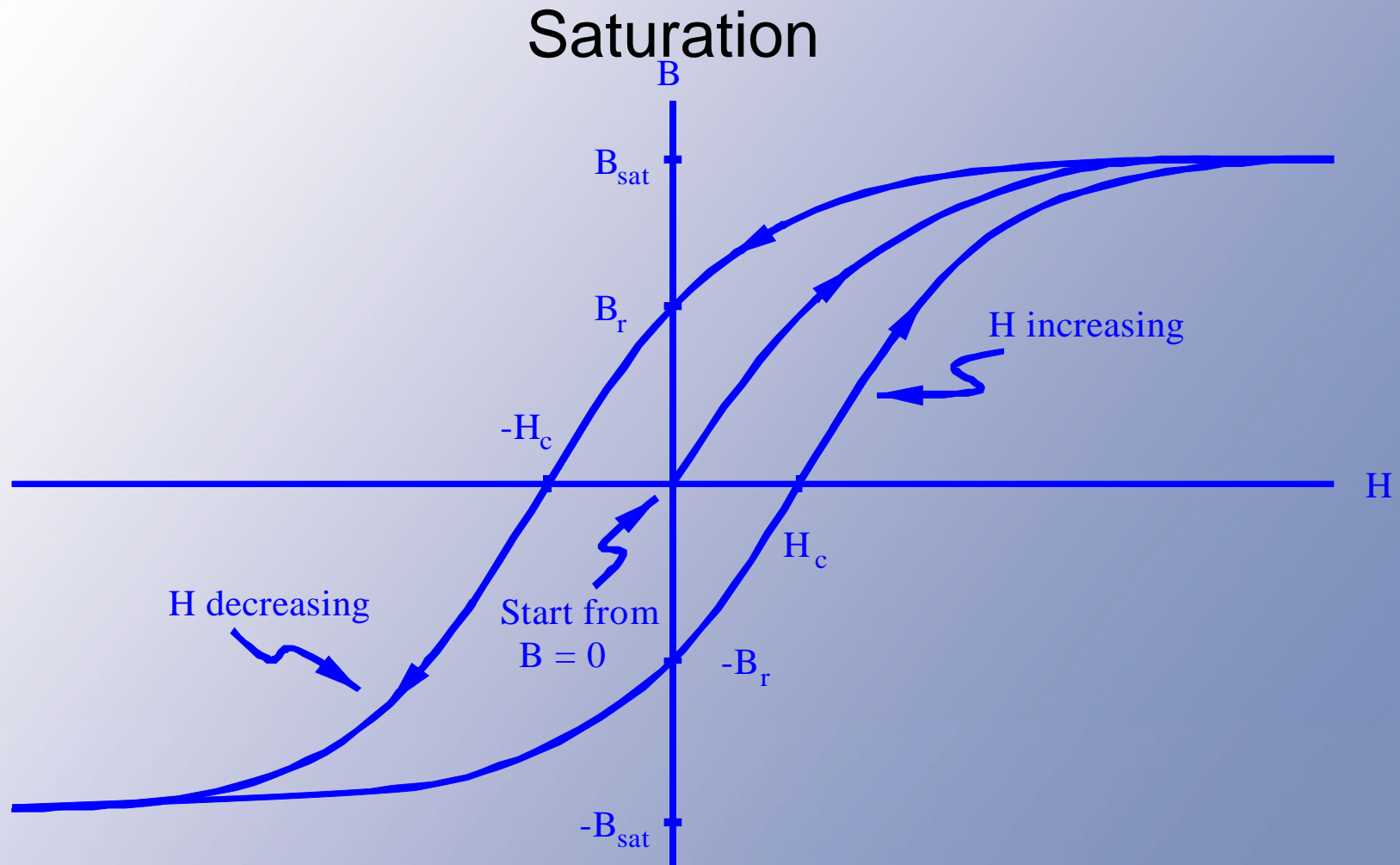
- Loss in the core is given by $P_{\text{loss}} = P_0 f^\alpha B^\beta$, where α and β are material constants (from measurements).
- Ideally α should be between 1 and 2 and $\beta = 2$, but the reality is a bit worse.
- Example: 3F3 ferrite material has $\alpha = 1.4$, $\beta = 2.4$.



Saturation

- What happens in saturation? At high B levels, the permeability drops toward μ_0 .
- The core becomes “transparent” and acts like the surrounding air.
- Little additional energy is stored.







Saturation

We avoid saturation for at least three reasons:

1. If the core is invisible, why use it?
2. With little extra stored energy, saturation is not helpful.
3. The inductance drops a lot, and usually currents rise excessively.





Saturation and Flux

- For steel and iron, the saturation level exceeds 1.5 T (1.5 Wb/m²).
- For ferrites, the saturation level is about 0.35 T.
- In design, we often use 1 T and 0.3 T, respectively.
- Therefore, we have a definite value B_{sat} that must be considered in design.



Finding Flux Density

- For design, we want $B < B_{\text{sat}}$.
- The flux is $\text{MMF}/\text{reluctance}$.
- In a single-winding core, $\phi = Ni/\mathcal{R}$.
- Flux density $B = \phi/A$, $B = Ni/(\mathcal{R}A)$.
- We want $B < B_{\text{sat}}$, so $Ni/(\mathcal{R}A) < B_{\text{sat}}$.
- There is an MMF limit:

$$Ni < B_{\text{sat}}\mathcal{R}A.$$





Amp-Turn Limit

- So we have an amp-turn limit, $Ni < B_{\text{sat}} \mathcal{R}A$.
- With $\mathcal{R} = \ell/(\mu A)$, this gives $Ni < B_{\text{sat}} \ell/\mu$.
- What are the implications for energy storage?
 $\frac{1}{2} L i^2$ now is limited since current has a limit.



Energy Limit

- Since $L = N^2/\mathcal{R}$, the stored energy $\frac{1}{2} Li^2$ is given by $\frac{1}{2} N^2i^2/\mathcal{R}$.
- At maximum Ni, we have $W_{\max} = \frac{1}{2} (Ni)_{\max}^2/\mathcal{R}$, with $Ni_{\max} = B_{\text{sat}}\mathcal{R}A$.
- Therefore $W_{\max} = \frac{1}{2} B_{\text{sat}}^2 \mathcal{R}A^2$.
- Simplify: $W_{\max} = \frac{1}{2} B_{\text{sat}}^2 \ell A/\mu$.



Energy Limit

- Interesting: Higher reluctance (lower permeability) leads to higher energy.
- Since ℓA is the core volume, storage is proportional to volume.

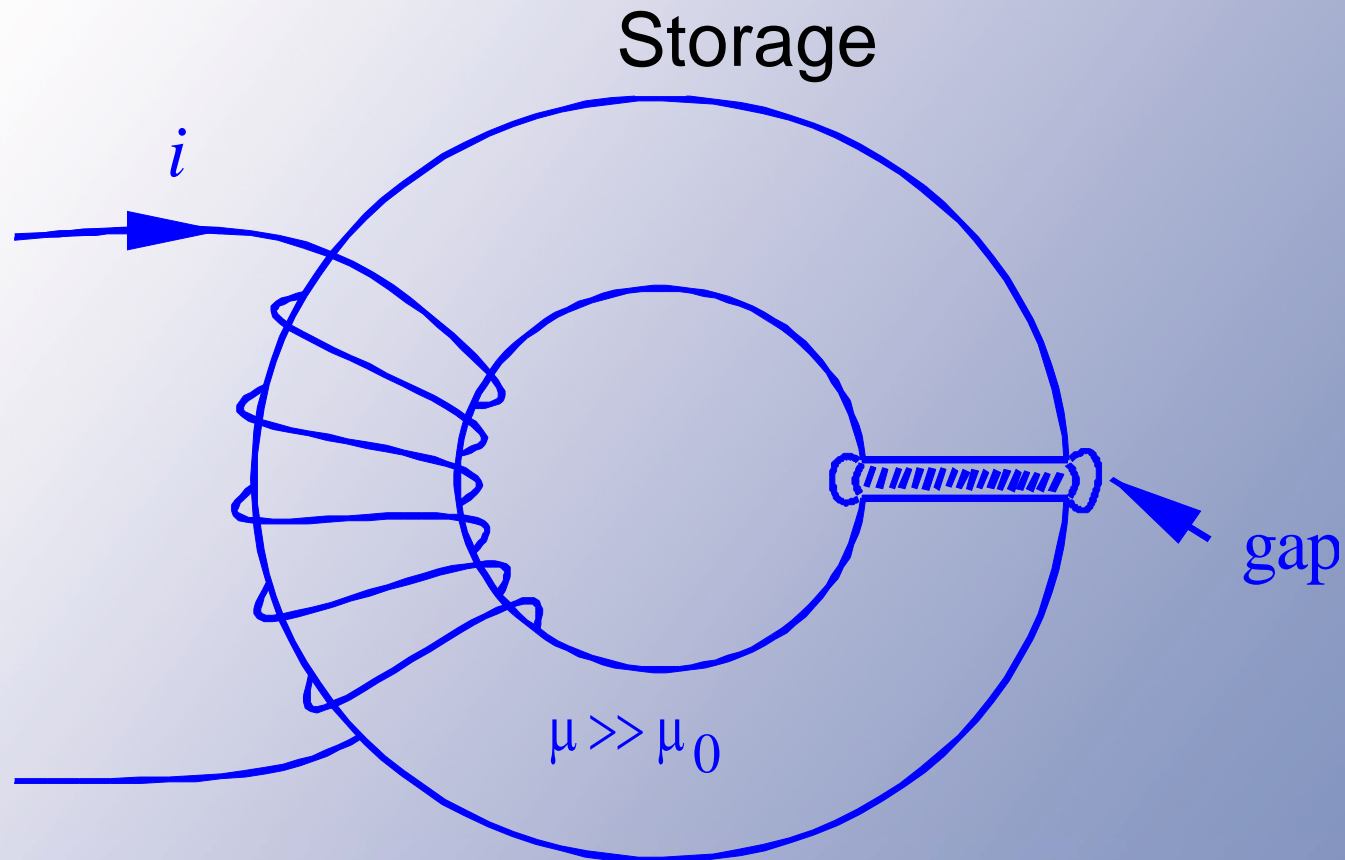




Storage

- Inductors nearly always have air gaps, which act as the energy storage region.
- The maximum energy relation can be used to determine air gap volume, and to estimate total core volume.





- Since $\mu \gg \mu_0$, almost all energy storage is inside the gap volume.



Storage

- Stored energy in an air gap:
$$W_{\max} = \frac{1}{2} B_{\text{sat}}^2 \text{Vol}_{\text{gap}} / \mu_0.$$
- Consider 1 mH, 20 A inductor with a ferrite core. Then $B < 0.3 \text{ T}$.
- The target stored energy is 0.2 J.
- The gap volume should be
$$(0.2 \text{ J})(4\pi \times 10^{-7} \text{ H/m})(2)/(0.3 \text{ T})^2 = 5.59 \times 10^{-6} \text{ m}^3.$$
- The gap volume should be about 6 cm^3 .



Storage

- Since the gap volume is a small fraction of the core volume, this translates to a core that is many cm^3 in size.
- We could just use an air core, but this makes all the flux leakage flux, and couples it into the outside world.
- It is hard to make substantial L values with an air core.





Design Issues

- Two design issues so far:
 - An amp-turn limit.
 - A gap volume for energy storage (which also implies a core volume).



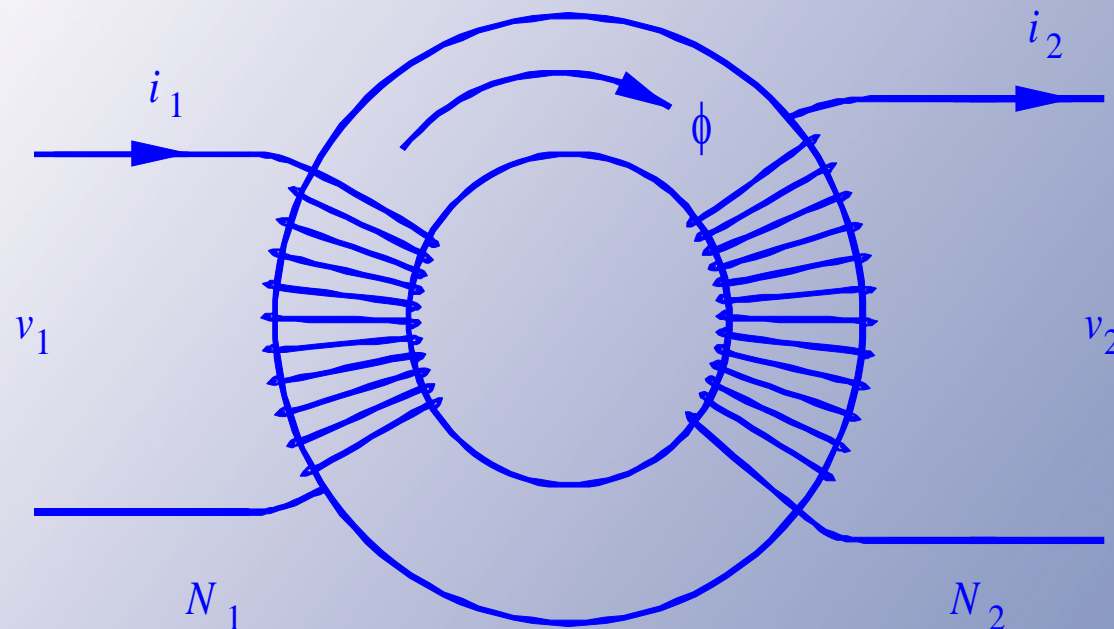


Other Limits

- In transformers, the amp-turn limit is not really useful, since there are multiple windings acting together.
- In any case, $v = d\lambda/dt$, so $\lambda = \int v dt$.
- This is related: $\lambda = N\phi = NBA$.



Transformer Limits



- Given ϕ , $v_1 = d\lambda_1/dt = N_1 d\phi/dt = N_1 A dB/dt$.
- So $\int v_1 dt = N_1 AB$, $B < B_{\text{sat}}$.



Volt-Second Limit

- The integral $\int (v dt)/(NA) < B_{\text{sat}}$ represents a *volt-second limit*.
- For square waves, with piecewise-constant voltage, this is clear:
 $(V \Delta t)/(NA) < B_{\text{sat}}$, or $V \Delta t < B_{\text{sat}} NA$.
- For sinusoidal voltages, $v = V_0 \cos(\omega t)$; the integral becomes $V_0/(\omega NA) < B_{\text{sat}}$.
- The limit $V_0/N < \omega B_{\text{sat}} A$ is called a *volts per turn limit*.



Limits So Far

- All these limits reflect a single issue: maintain $B < B_{\text{sat}}$.
- The implications include:
 - A current limit, $Ni < B_{\text{sat}} \mathcal{R}A$.
 - A volt-second limit, $\int (v \, dt) < B_{\text{sat}} NA$.
 - An energy limit, $W < \frac{1}{2} B_{\text{sat}} \text{Vol}/\mu$.

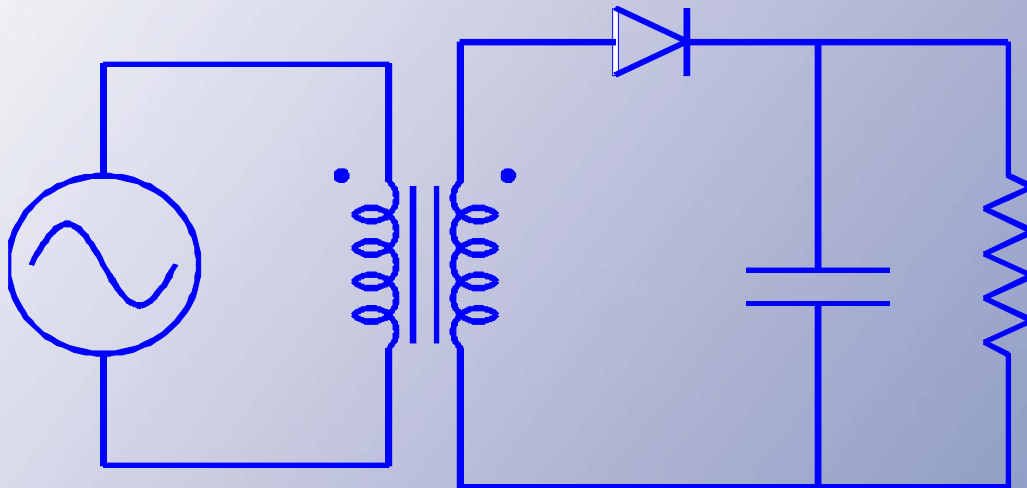


Current and Volt Limits

- A transformer that sustains dc current must satisfy both the amp-turn and volt-second limits.
- This is because dc current only acts in one winding.
- $\int (v \, dt)/(NA) + Ni_{dc}/(\mathcal{R}A) < B_{sat}$.



Dc Current Limits



- Circuits like this half-wave rectifier impose dc current on a transformer winding.
- The current produces a “flux offset.”
- Less flux is available to handle the voltage.



Example

- A laminated steel core, with 1 cm^2 cross-section, is to be used for a 120 V to 12 V, 60 Hz transformer. The Ampere's Law path length is 12 cm, and $\mu = 10^4 \mu_0$.
- How many turns?
- How much dc current can flow?



Example

The volt-second limit for $170\cos(120\pi t)$ V tells us $170/(\omega NA) < B_{\text{sat}}$.

For steel, let us keep $B < 1$ T. Then $N > 4510$ turns.

The limit is 37.7 mV/turn.



How Much Dc?

- Let us use 4510 to 451 turns for 120 V to 12 V. How much dc current is allowed on the low side?
- $Ni < B_{\text{sat}} \mathcal{R}A$, $\mathcal{R} = \ell/(\mu A) = 9549 \text{ H}^{-1}$.
- Ignoring the voltage, $Ni < 0.95 \text{ A-turn}$, and $i < 2.1 \text{ mA}$, but we cannot allow this much because of v !



Limits

- In general, it is hard for a transformer to tolerate dc current.
- 50 Hz and 60 Hz transformers are large and have many turns.
- A 6000 Hz transformer of the same size would need only 46 turns on the high-voltage side.
- Here perhaps 50:5 to get the desired 10:1 ratio.



Limits

The volts per turn limit suggests more turns.

The amp-turn limit suggests fewer turns.

We recall that wire size issues also lead to current limits.

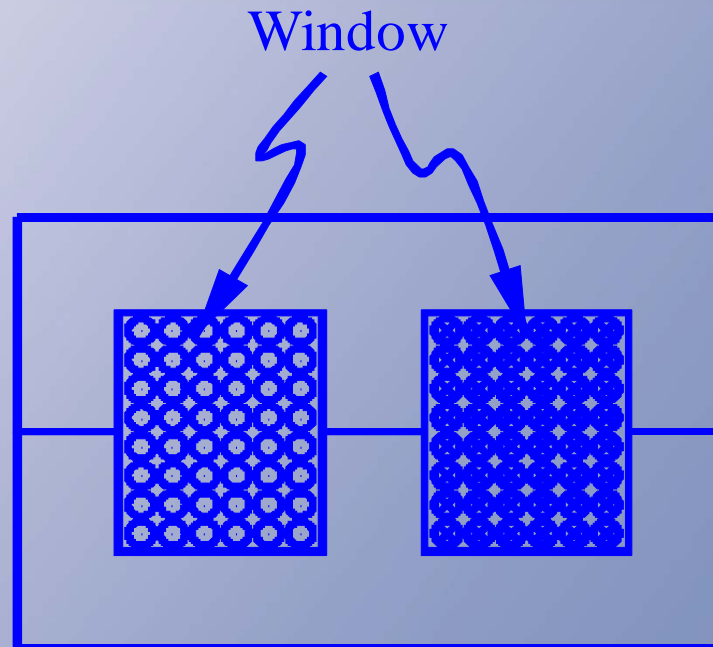
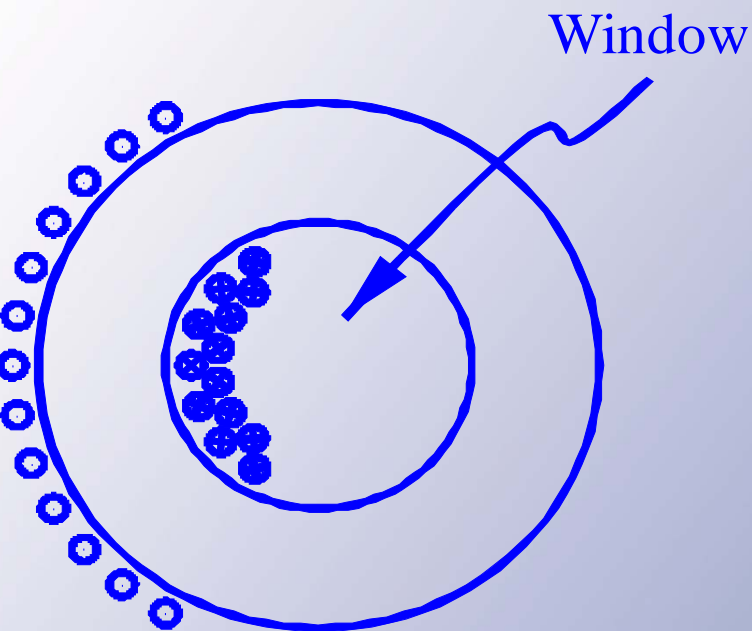


Copper Limit

- There is a geometric limit on copper: The amount of space for windings in any core.
- The windings pass through the core *window*.
- Not all the window can be filled with copper (insulation, air).
- The window has a *fill factor* less than 1.



Copper Limit



Toroid, fill factor ~ 10%

E - E core, fill factor ~ 50%

It is hard to have a window for which more than 50% is actual copper.



Copper Limit

- If we could fill 50% of the window, then the fill factor is $\frac{1}{2}$ and the copper area is $A_{\text{win}}/2$.
- This area carries Ni. It must meet the copper current density limit.
- $2\text{Ni}/A_{\text{win}} < J_{\text{limit}}$, which might be 100 A/cm^2 or so.
- This is another limit on amp-turns.



Design for Losses

The total losses are minimized if:

- Losses in windings match losses in the core.
- Losses match in each winding if there are several.

To match winding losses: Use the same current density J in every winding.



Winding Issues

Toroids are nice because they are easy to fabricate and good at confining flux.

However, they are hard to wind (usually wound by hand).

Other “split” cores are used for ease of winding.



Inductor Example

Design a 500 μH inductor that can handle 10 A (peak) in a 50 kHz PWM application.

Since there is one winding and the peak current is known, we can find flux from Ni/\mathcal{R} .



Material, Size?

- Choices: ferrite or powdered iron. Steel is probably not an option at 50 kHz.
- Powdered iron has higher saturation flux – and it is cheap.
- Let us select powdered iron (this is a possible frequency for it, although a touch high, but other powdered materials can achieve it).



Material, Size?

- This material has a “distributed air gap,” so total core volume is at issue.
- There are many kinds. Typical power parts have $\mu \approx 75\mu_0$.
- Our inductor must store up to $\frac{1}{2} Li^2 = \frac{1}{2} (500 \text{ uH})(10 \text{ A})^2 = 25 \text{ mJ}$.



Size

With saturation at 1 T, our energy limit requires
 $25 \text{ mJ} < \frac{1}{2} (1\text{T})^2 \text{Vol} / (75\mu_0)$.

The volume should be at least
 $4.7 \times 10^6 \text{ m}^3$, or 4.7 cm^3 .



Size

- In a catalog, I found a toroid with these properties, and volume of 6.44 cm^3 . The OD is 3.6 cm, the ID is 2.2 cm, and the core is 1.05 cm thick.
- Reluctance? Length = 9.16 cm (from the catalog), $A = 0.711 \text{ cm}^2$.
- Winding window? The window diameter was 2.2 cm, for an area of 3.94 cm^2 .



Reluctance

The reluctance is $1.37 \times 10^7 \text{ H}^{-1}$, from these numbers.

The manufacturer reports $A_L = 73 \text{ nH}$, which is consistent.

We want $N^2 \times A_L = 500 \text{ uH}$, so $N^2 = (500 \text{ uH}) / (73 \text{ nH}) = 6849$.

This gives $N = 83$.



Coil

Will the wire fit? We want 10 A. For powdered iron, the density should be in the 500 A/cm^2 range (or less) for loss matching.

This is equivalent to #14 AWG, although we will probably strand it, since #14 is hard to wind.



Coil

- #14 wire has an area of 2.08 mm^2 .
- For 83 turns, this is 173 mm^2 .
- The window diameter was 2.2 cm, for an area of 3.94 cm^2 , or 394 mm^2 .
- This requires 44% fill. The window might be just big enough, but this is very tight.
- A larger core would be better: this one pushes thermal limits.



Larger Core

- A larger core: T200-26 toroid. This has an outside diameter of 2.0 in (50.8 mm) and inside diameter of 31.8 mm.
- Catalog information: volume 16.4 cm^3 , magnetic path length $\ell = 13.0 \text{ cm}$, core cross section area $A_e = 1.27 \text{ cm}^2$.
- The catalog lists $A_L = 92 \text{ nH}$.



Larger Core

- To get 500 μH , we need $N^2/\mathcal{R} = N^2 A_L = 500 \mu\text{H}$.
- This requires $N^2 = 5435$, $N = 74$ turns.
- The window is much larger, at 794 mm^2 .
- This is a 22% fill. Now the wire fits better.
- We can move to #12 wire, which gives reasonable current density.



Flux

- We apply 740 A-turns to the core.
- The flux is Ni/\mathcal{R} , or $740/1.09 \times 10^7$. This is 6.8×10^{-5} Wb.
- From the area of 1.27 cm^2 , the flux density is $B = 0.53 \text{ T}$.
- Not close to saturation, and this should help keep losses low.
- The toroid design is a powdered iron core with 74 turns of #12 AWG wire.



Transformer Example

- How much power can a ferrite toroid with 0.87 in OD, 0.54 in ID, and 0.25 in thickness handle in a forward converter application? The switching frequency is 200 kHz, and the peak voltage is 200 V. The turns ratio is 10:1.
- Notice that a ferrite has been selected because of the high frequency.



Transformer

- We want a core with low reluctance because this is a transformer.
- The core must handle 200 V for 2.5 μ s, so the volt-second rating must be at least 500 μ V-s.
- From our integration, the volt-second product gives $V\text{-s}/(NA) < B_{\text{sat}}$.
- For this core, $A = 0.259 \text{ cm}^2$. We need $N > 64$ turns on the high-voltage side.
- Use 70:7 to give 10:1 ratio.



Windings

- There are two windings, and also 50% fill at best.
- Each winding should have the same current density to be sure neither is overdesigned.
- Therefore, each winding takes up the same total area.
- Thus let the primary copper take up no more than 25% of the window.



Windings

- The window area is 1.4 cm^2 , so the winding copper can use up to 0.35 cm^2 . With 70 turns, this is 0.50 mm^2 per turn.
- If we choose a wire size of #21 AWG, it has an area of 0.41 mm^2 . This should fit.
- At 250 A/cm^2 , it can carry 1 A.



End Result

- We have designed as follows:
 - Primary: 70 turns of #21 AWG, rated for up to 1 A.
 - Secondary: 7 turns of #11 AWG, rated for up to 10 A.
 - The primary can handle a 200 V square wave.
 - The secondary can handle a 20 V square wave.



Result

- This small core can handle 200 V peak and 1 A of flow, with a square wave input. This gives 200 W.
- A small core less than 1 inch across provides a 200 W transformer!
- The same core at 60 Hz would need 217000 turns and would handle 60 mW – if we could find small enough wire.

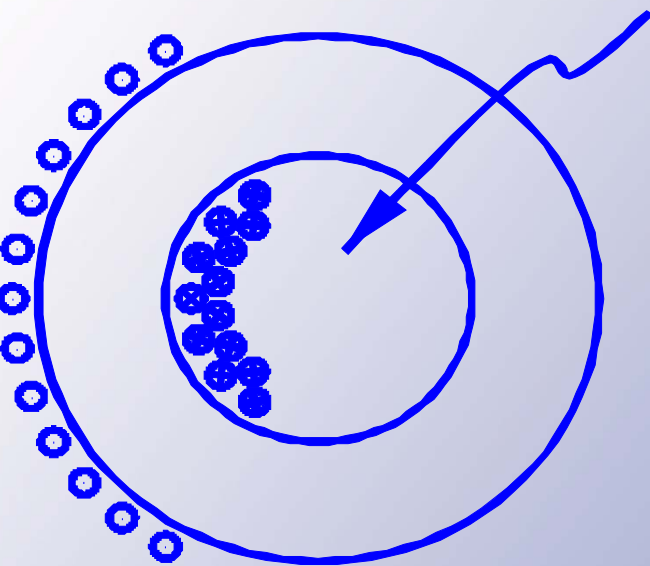


Core Types

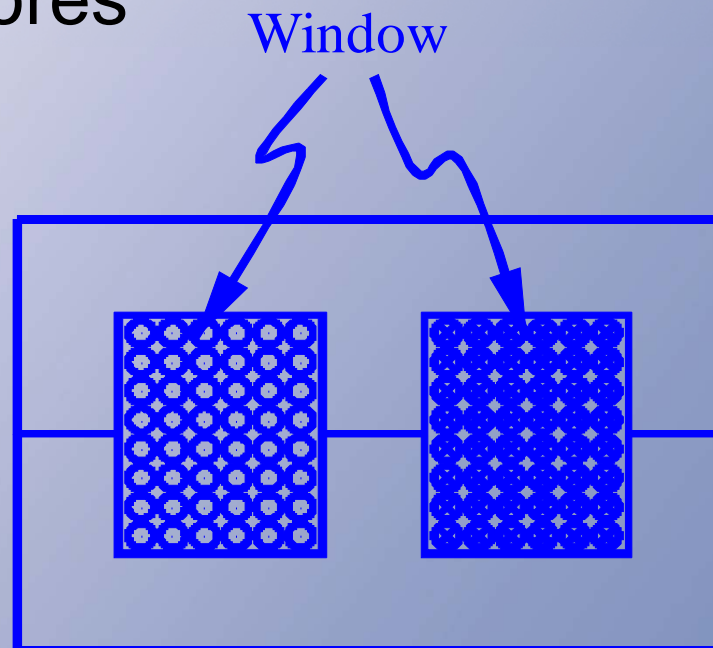
- Toroids are common for cases with just a few turns.
- They confine fields well, and require little extra shielding.
- The shape tends to give a low cost.
- But, it is hard to wind them.
- Also, a change in permeability requires a change in materials.



E-E Cores



Toroid, fill factor $\sim 10\%$



E - E core, fill factor $\sim 50\%$

- E-E cores are one of the simplest “split core” types.



E-E cores

- With a split core, the winding can be set up on a separate bobbin or paper tube, then the core is assembled around it.
- This leads to very simple machine-wound coils.
- With a split core, an air gap can be produced by grinding down the center post or adding a filler sheet.



E-E cores

- E-E cores are in common use for transformers and often for inductors.
- High permeability materials are used most often, and any change involves the air gap rather than the material.
- Most E-E cores are either steel laminations or cast ferrites.



Pot Cores

- A pot core looks like an E-E core that has been rotated.
- This geometry gives extremely good magnetic shielding.
- As in the E-E core, the winding is set up on a separate bobbin.
- The center post can be ground to make an air gap.





Pot Cores

- Pot cores work well in the highest frequency applications.
- They are more expensive because of the complicated shape. Ferrites are used.
- Often, the current density must be lower for pot cores, since the winding is enclosed.