Power Electronics

Day 8 -- Components

P. T. Krein
Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign

Quasi-Steady Loads

- Examples are any load that does not change much during a switching period.
  - Motors
  - Loudspeakers
  - Batteries to be charged
  - Almost any load that includes a filtering effect

Quasi-Steady Loads

- We often make loads quasi-steady by adding series inductance or parallel capacitance so they will not change rapidly.
- From the converter, such loads act – on short time scales – like ideal sources.

Transient Loads

- Some loads change so fast that switch action does not do much.
- Good example: large microprocessor.
  - A processor with a 1000 MHz clock can change its current needs every 1 ns.
  - A lot of change can occur in between switching actions inside the power converter.

Transient Loads

- Transient loads must be addressed through energy storage, since switch action is not sufficient to provide the correct voltage or current.
- The energy storage needs become part of the power converter design.

Transient Loads

- Example: A certain microprocessor can be modelled as a capacitance (inside the CMOS chip) switching into the input voltage at 1000 MHz.
- Consider a 10000 pF capacitor switching into a 1 V source at this rate.
- Each time we switch, charge CV = 10000 pC must be delivered. At this rate, this is 10 A of current (or 10 W).

Transient Load Example

- With a 5 V to 1 V converter, the diode carries the load current most of the time.
- The concept: provide enough output capacitance that the load effect will make little difference.
- For example, a 10 uF capacitor should be able to charge 0.01 uF about 100 times before the voltage falls 10%.

Transient Loads

- This gives us 100 ns rather than 1 ns until the next switching is needed.
- This is still a power converter running at about 10 MHz.
- The reality is that even more capacitance is needed.

Source and Load Limitations

- For *any* real source or load, connection wiring is required.
- Connections always introduce resistance, but also inductance.
  - The inductance arises because any current generates a magnetic field.
  - The interaction between the field and the circuit is modelled as inductance.

Current Sources

- Fundamentally, this means all sources and loads ultimately act like current courses, at least at short enough time scales.
- We originally claimed that voltage and current sources are about equally common, but the reality is that everything is a current source (on a fast scale).

Current Sources

- At a fast enough time scale, the inductance will always be greater than the critical value.
- We will need to get an idea of wire inductance effects and evaluate their impact on switching.

Wire Inductance

- A wire has self-inductance, since the current in it generates a magnetic field that interacts with the return conductor.
- The field also interacts with part of the current inside the wire.

Wire Inductance

- There is internal inductance owing to the field inside the wire.
- External inductance is the interaction between field outside the wire and the current.
- Both are self-inductance, rather than mutual inductance.

Wire Inductance

- The self-inductance of a long wire is a classic electromagnetics problem.
- L (in henries) per unit length is $\mu_{wire}/(8\pi) + \mu/(2\pi)$ ln D/R.
- D is the center-to-center distance between the wire and the return conductor. R is wire radius.
- The value μ is the permeability.

Wire Inductance

- The first term, $\mu_{wire}/(8\pi)$, is the *internal self-inductance*.
- For aluminum, copper, silver, gold, $\mu_{wire} = \mu_0 = 4\pi \times 10^{-7}$ H/m.
- For steel or nickel, it is much higher.
- For μ_0, the term is 50 nH/m.

External Inductance

- The external term $\mu/(2\pi)$ ln D/R depends on μ of the insulating material. This is usually μ_0.
- The spacing is not a very sensitive function.
- For D/R = 10, the value is about 460 nH/m.
- Add the internal effect, yields about 500 nH/m

Self-Inductance

- For copper wire and air, plastic, or varnish insulation, we have:

D/R	L	
100	950 nH/m	
10	500 nH/m	typical
3	250 nH/m	minimum

- For wire, 2 nH/cm is a lower bound.

Wire Inductance

- But, we need two wires (the second for return).
- Inductance is minimized when the wires are close together.
- Why? This tends to cancel external magnetic fields.

Wire Inductance

- For a wire pair, 10 nH/cm is a typical value.
- 4 nH/cm is a dead minimum for a very tightly twisted pair.
- How to reduce these?
  - The ultimate twisted pair: Litz wire.
  - Bus bar.

Implications

- Think about a converter -- built with wires.
- Example: Buck converter with 10 cm of wire between source and switches.
- This gives about 100 nH total – 50 nH in each leg.

Switching Effects

- KCL problem! If we switch current instantly, the inductance generates infinite voltage.
- The saving factor is that switches take time to operate.
- What if 10 A is switched in 40 ns?

Switching Effects

- Then $v_L = L\, di/dt \approx 100$ nH(10A)/(40 ns).
- The inductor voltage is 25 V.
- Now think about 100 cm of wire and 50 A of current -- 1250 V!

Effects

- Effects include
  - Time delay from source to input
  - The switches see a much higher than expected voltage.
  - Ground reference node – where should it be? Ground bounce.
  - Voltage tolerance. The extra inductor voltage introduces ripple and error.
  - Extra losses and KCL problems.

Example

- Example: A boost converter delivers its square wave to a capacitive load through 2 cm of wire.
- The square wave is 20 A in amplitude, and the switching requires just 50 ns with a switching frequency of 250 kHz.
- If we estimate based on $v_L = L\,di/dt$, we get $v_L \approx (20\text{ nH})(20\text{ A})/(50\text{ ns}) = 8\text{ V}$.

Example

- Large impact of inductance on ripple waveforms.
- Real systems also show ringing and resonant behavior as well.

Critical L and C

- We know that $L > L_{crit}$ assures both $i_L > 0$ and $\Delta i < \pm 100\%$.
- Similarly for C.
- In general, the ratio L/L_{crit} serves as a measure of quality.
- If $L = L_{crit}$, we have a current source, but it is not ideal.
- $L >> L_{crit}$ defines an ideal current source.

Critical L and C

- Since critical L and C are usually easy to compute, we can determine how much L or C to provide to make a source or load "ideal."
- This is another approach to the "interface problem:" add passive storage elements to make a real source or load more nearly ideal.

Dc Source Interfaces

- To form a dc current source, simply add a series inductance (well) above the critical value.
- To form a dc voltage source, add parallel capacitance (well) above the critical value.

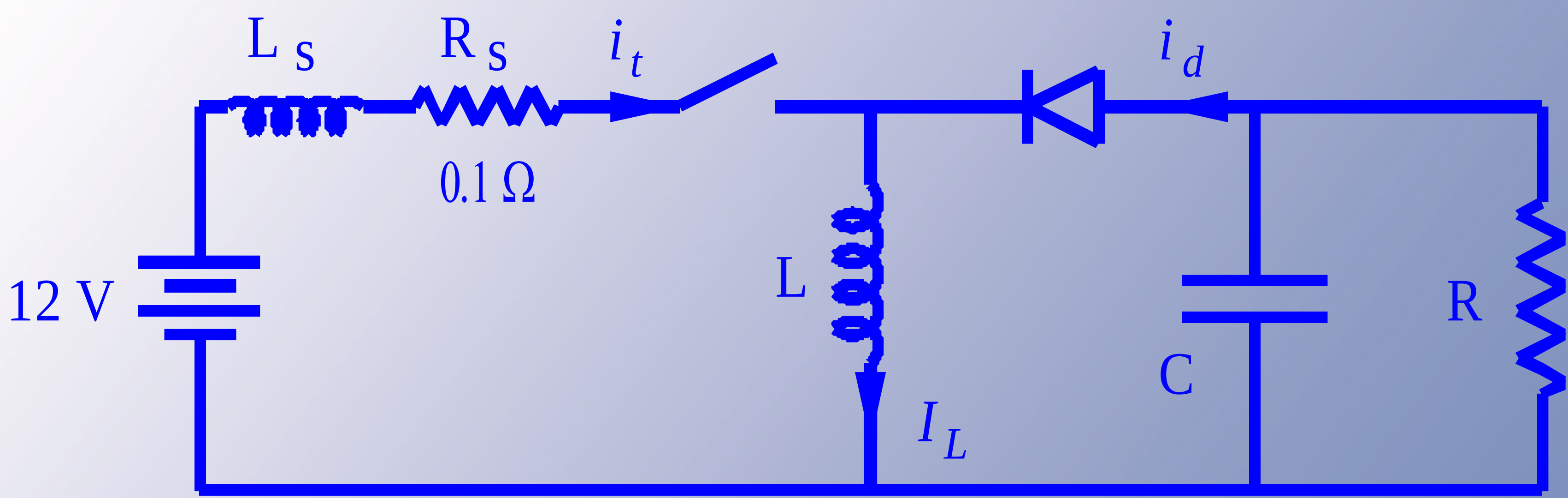
Example: Battery Source

- A battery has series L and R.
- Even ignoring the L, the resistance carries current part of the time: $P_{loss} = D\, I_L^2\, R_s$.
- With an interface capacitor, the battery sees current $(D\, I_L)$ instead.
- The loss is $(D\, I_L)^2\, R_s$, which is lower by a factor of D.

Interface Example

- A battery with internal resistance of 0.1 Ω supplies a buck-boost converter.
- The load is 200 W, and the nominal conversion is +12 V to -12 V.
- 50 kHz switching; $L >> L_{crit}$.

Interface Example



- Let L >> L_{crit}, C >> C_{crit}, f_{switch} = 50 kHz.
- Ignore L_s for now.
- Voltage drop: $i_t R_s = q_1 I_L R_s$

Analysis

- The load current is (200W)/(12V) = 16.7 A.
- Notice that the transfer voltage is $v_t = q_1(V_{bat} - I_L \times 0.1\ \Omega) + q_2(V_{out})$.
- The diode current is $i_d = q_2 I_L$.
- Averages: $I_{load} = D_2 I_L = 16.7$ A;
  $\langle v_t \rangle = D_1(V_{bat} - 0.1 I_L) + D_2 V_{out} = 0$;
  $D_1 + D_2 = 1$.

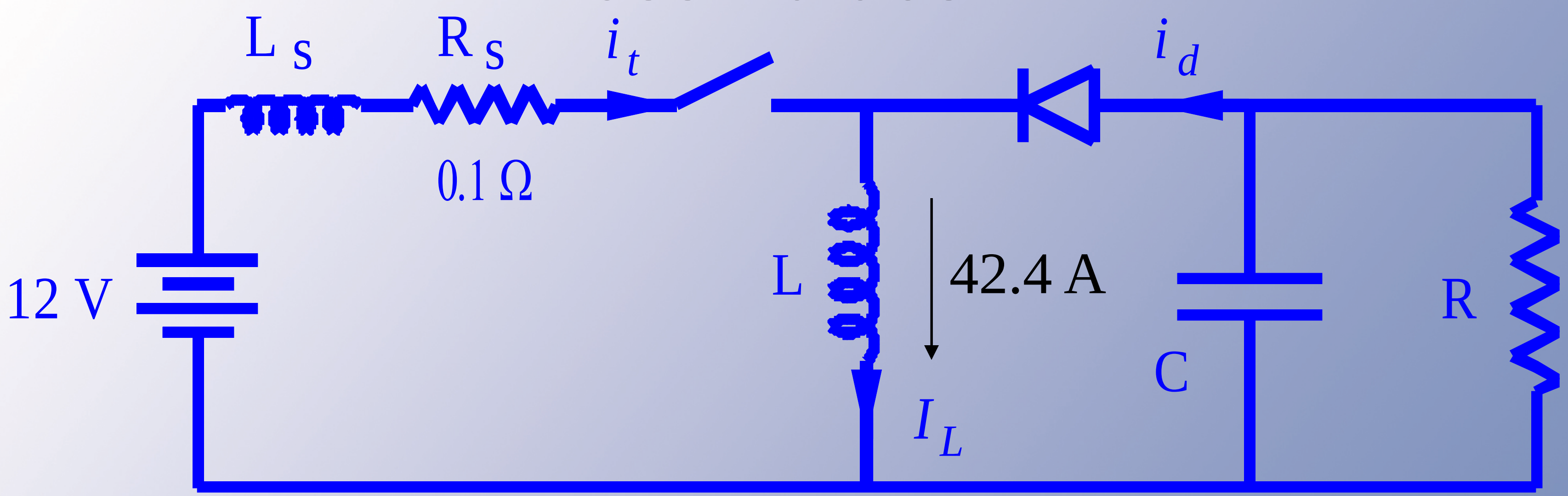
Analysis

- This gives three equations in the unknowns D_1, D_2, and I_L.
- Combine to give:
  $12D_1 - 16.7(0.1)D_1/(1 - D_1) - 12(1-D_1) = 0.$
- Two solutions: $D_1 = 0.607$ (the correct one), $D_1 = 0.823$ (a high-loss answer).

Loss Values

- With this result, I_L = 42.4 A.
- The loss with no interface is I_L^2(0.1 Ω) while switch #1 is on.
- The average power loss is D_1(42.4 A)^2(0.1 Ω) = 109 W.
- Efficiency is 64.7%.

Loss Values

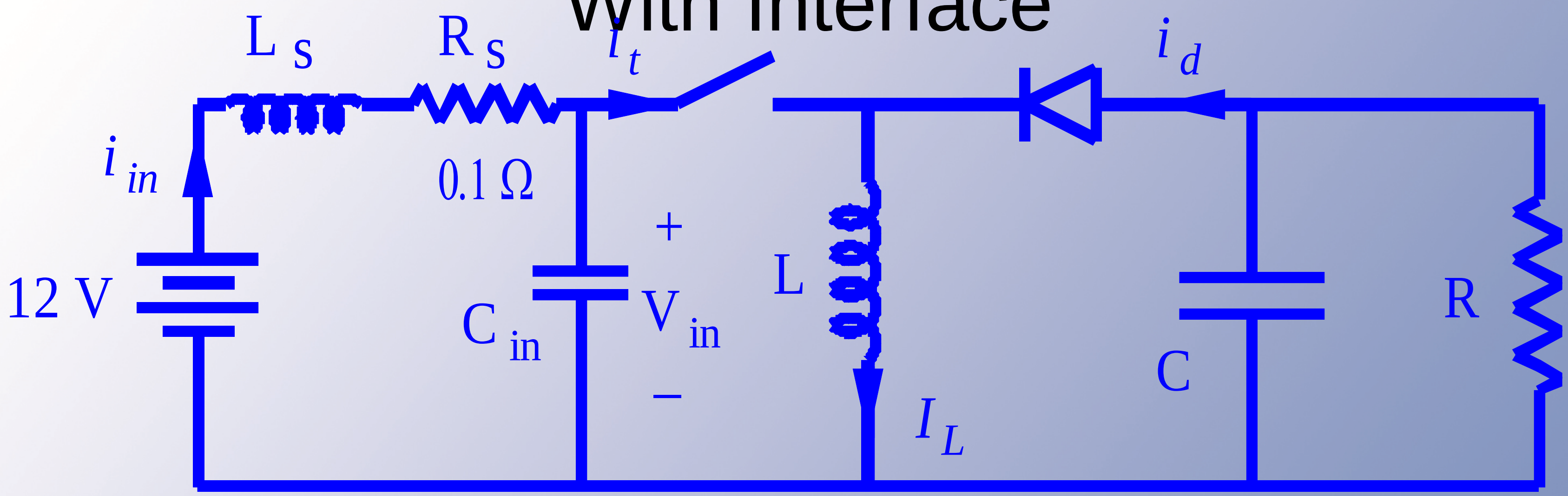


- $P_{loss} = D_1 (42.4^2 R_s) = 109$ W
- Pin = 309 W, $\eta = 64.7\%$.

Now, an Interface

- Instead, let us provide a large capacitor at the battery terminals.
- Now the battery is exposed to the average input current instead of the inductor current.
- The transfer voltage is $v_t = q_1(V_{bat} - I_{in} \times 0.1\ \Omega) + q_2(V_{out})$.

With Interface



- C_{in} is the source interface.
- Now $i_t = q_1 I_L$, but $I_{in} = \langle i_{in} \rangle = \langle i_t \rangle = D_1 I_L$.
- $V_{in} = V_{bat} – I_{in} R_s = V_{bat} – D_1 I_L R_s$.

With Interface

- $v_t = q_1(V_{bat} – D_1\ I_L\ R_s) + q_2(V_{out})$.
- $\langle v_t \rangle = 0 = D_1\ (V_{bat} – D_1\ I_L\ R_s) + D_2\ V_{out}$
- Also, $i_d = q_2 I_L$, and $\langle i_d \rangle = I_{load} = 16.7$ A.
- Then $0 = D_1(V_{bat} – D_1/D_2\ I_{load}\ R_s) + D_2 V_{out}$

With Interface

- Is this really any different? Now, $12D_1 - 16.7(0.1)D_1^2/(1 - D_1) - 12(1-D_1) = 0$.
- The solutions are $D_1 = 6/11$ (or 0.545) and $D_1 = 6/7$ (or 0.857). The first is correct, since the second involves high loss.
- $I_L = (16.7\ A)/D_2 = 36.7\ A$, and $I_{in} = 20\ A$.

Current and Loss

- The loss is I_{in}^2 R = 40 W.
- The efficiency is 83.3%.
- We cut out almost 70 W of loss just by adding an interface.
- **The loss dropped by 64%**, just be adding one part!

→ Source interfaces are essential for good design.

Source Impedance

- Ideal voltage (dc or ac):
  - Definite v(t) function no matter what the current.
  - No loss (just output or input power).
  - No imposed current is associated with any voltage drop.
- This means Z = 0 (except that power flows at f_{source}).

Source Impedance

- The effect is well known: a dc voltage source acts like a short circuit to ac signals.
- It is also true that an ac voltage source acts like a short circuit (except at its own frequency).

Source Impedance

- Ideal current, ac or dc:
  - Definite current i(t) no matter what the voltage.
  - No loss (just power flow).
  - Any imposed voltage does not generate an associated current.
- $Z \rightarrow \infty$ (except for f_{source})

Real Sources/Loads

- Series or parallel resistance causes loss.
- Series L causes impedance to rise with frequency.
- If ac sources must handle dc voltage or current, special problems arise.

Dealing with Z

- For dc voltage, a parallel capacitor will make Z fall with increasing frequency.
- A capacitor makes the source more ideal in several ways.
- For dc current, series L makes Z rise, and the source is more ideal.

Ac Sources

- Consider ac voltage: We want $Z = 0$, except that the interface should not cause trouble at f_{source}.
- Parallel L-C can do this: If we set $1/\sqrt{(LC)} = 2\pi f_{source}$, then this interface has no effect at f_{source}, but can have $Z \to 0$ elsewhere.

Ac Sources

- For ac current, series L-C is appropriate.
- Once again, we should choose the resonant frequency to match that of the source.

General Cases

- If we can avoid subharmonics, then these reduce to parallel C and series L.
- If not, true resonant pairs might be necessary.

Traps

- True resonant pairs end up with large parts when the frequencies are low.
- So far, we have focused on eliminating *all* frequencies other than the wanted one – any value $f \neq f_{in}$.
- Although the number of unwanted components is infinite, often the unwanted frequencies are known.

Tuned Traps

- We can focus instead on the unwanted frequencies, and block them specifically.
- This is the idea of a tuned trap: make $Z = 0$ or $Z \to \infty$ for specific unwanted frequencies rather than for a whole range.

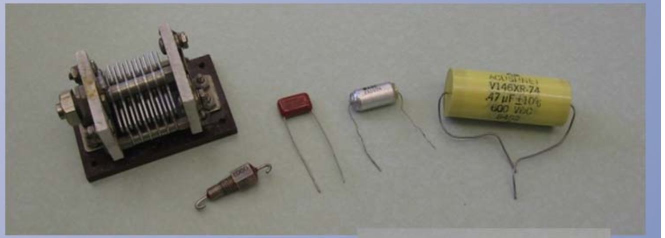
Rectifier

- Example: Source interface for a six-pulse rectifier.
- We know the currents contain harmonics that are odd multiples of the ac line input.
- Resonant interfaces can be added at key unwanted frequencies to help reduce them.

Summary so far

- For dc sources, inductors and capacitors well above critical values serve as interfaces.
- For ac sources, series L and parallel C work in restricted frequency ranges.
- For ac sources, resonant LC filters and traps can be used to create more ideal characteristics.

Capacitor Types



- Simple dielectrics:
  - Two conductive plates with a planar dielectric in between.
  - A wide variety of dielectric materials.
- Electrolytics:
  - The dielectric is formed electrochemically on a metal.
- Double-layer

In General

- We define electrical permittivity, ε, and $C = \varepsilon A/d$. A is the plate area, d is the plate spacing.
- The permittivity of free space is ε_0 = 8.854 pF/m.
- Large plate areas and small spacings are needed.

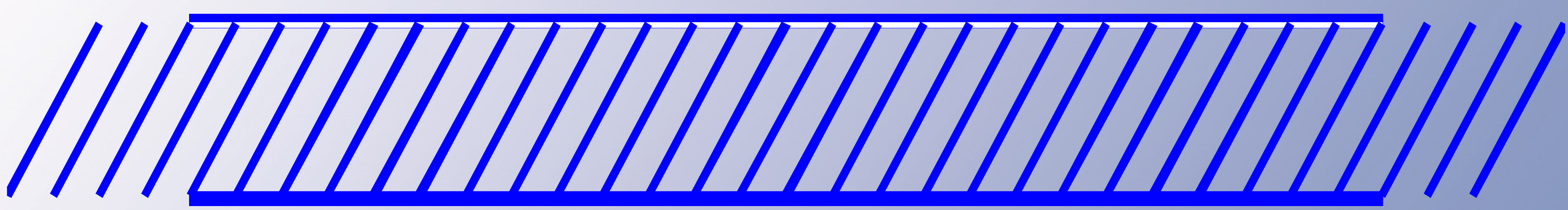
Voltage Limits

- Any capacitor has a voltage rating, determined by the dielectric breakdown strength.
- The electric field is V/d. Typically, the limit is 10 MV/m or so. For a typical 25 um dielectric, this gives 250 V or more.

Value Example

- It is difficult to build capacitors with large values.
- Example: Let A = 1 m^2, d = 5 um.
- Since $\varepsilon \approx$ 10 pF/m, the capacitance $C = \varepsilon A/d$ is about (10 pF/m)(1 m^2)/(5 um) = 2 uF.
- This is a big object for such a modest value.

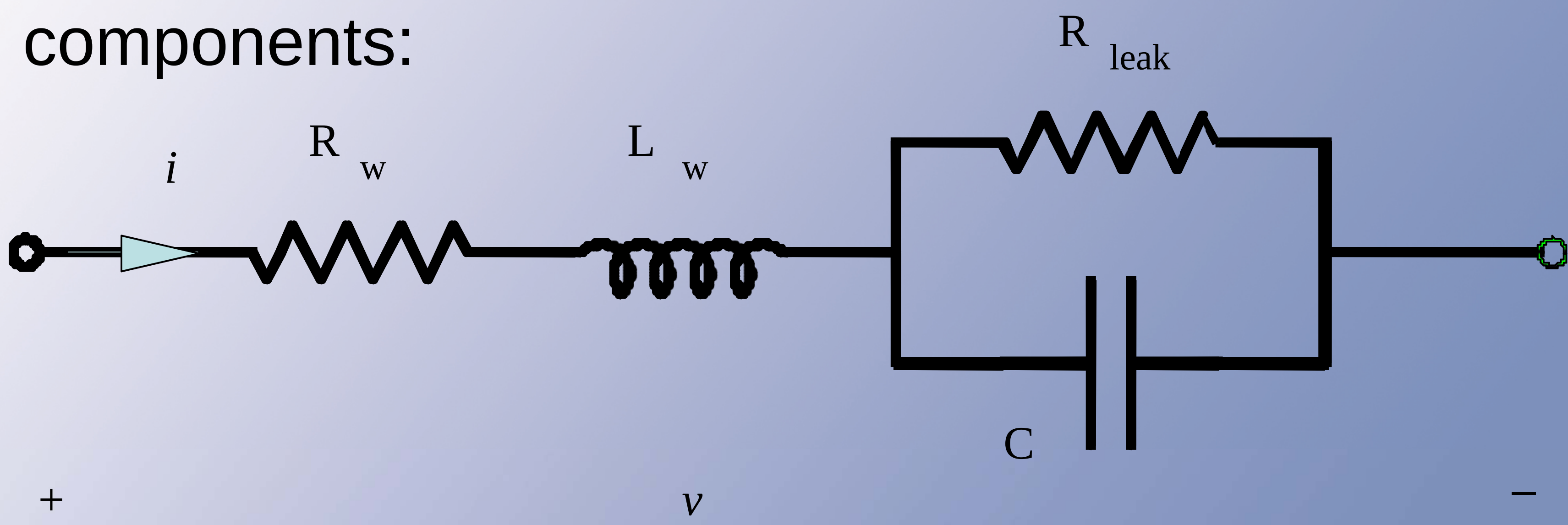
Simple Dielectrics



- Typical voltage ratings: 100 V and up.

Circuit Models

- Because of wires, connections, and resistance, a capacitor really involves several extra components:



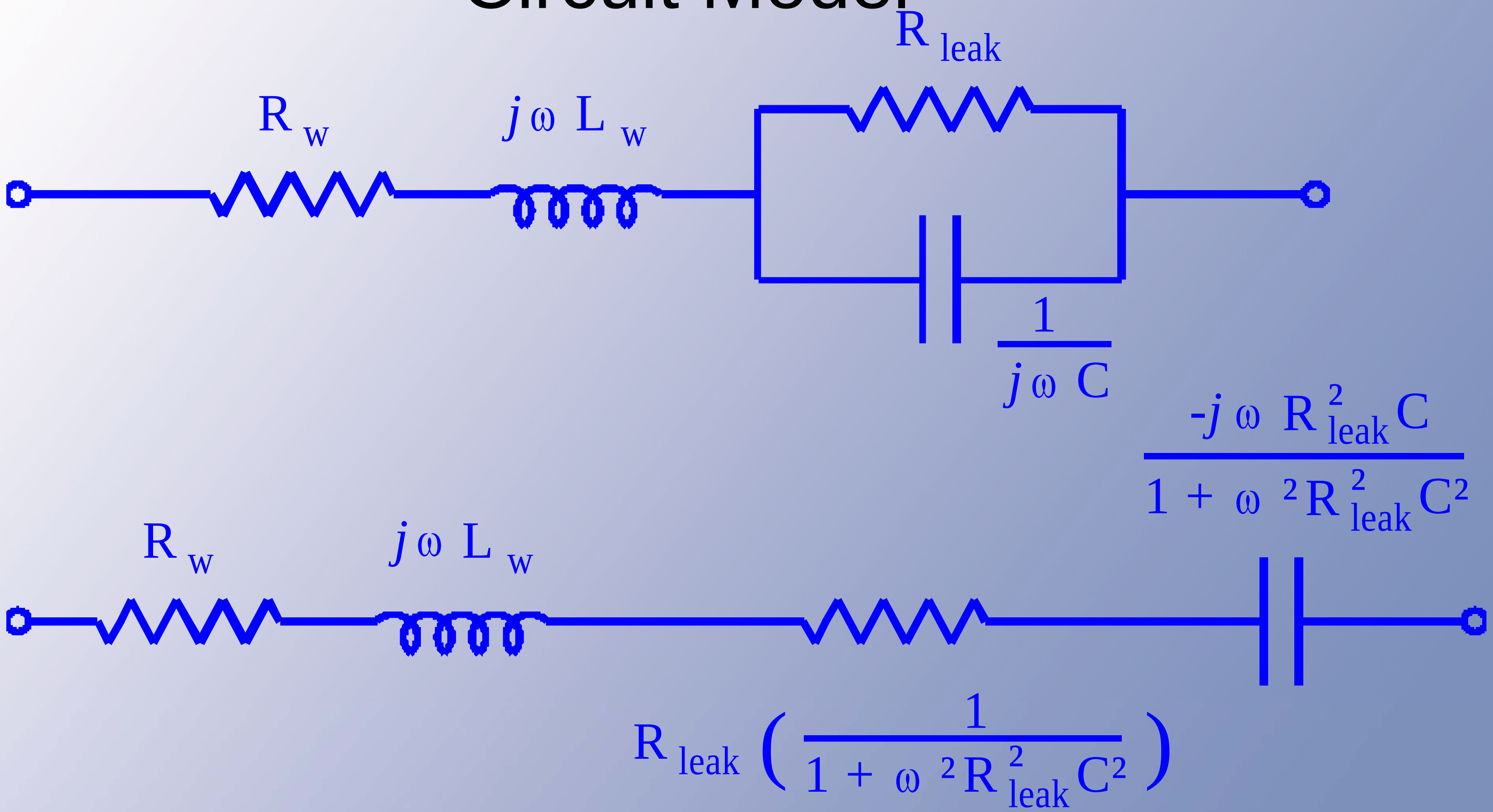
Circuit Model

- The wire must show resistance and inductance.
- The insulator should have some leakage resistance.
- In a converter, we should consider large unwanted components to understand capacitor action.

Circuit Model

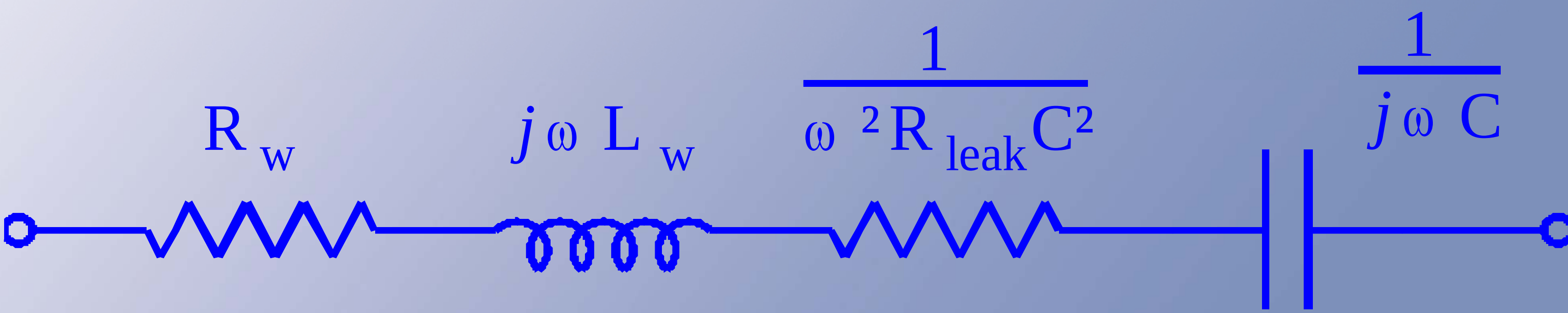
- Focus on a single radian frequency ω.
- The parallel RC can be reduced to a series equivalent.
- We are left with R, L, and C as a series combination.

Circuit Model



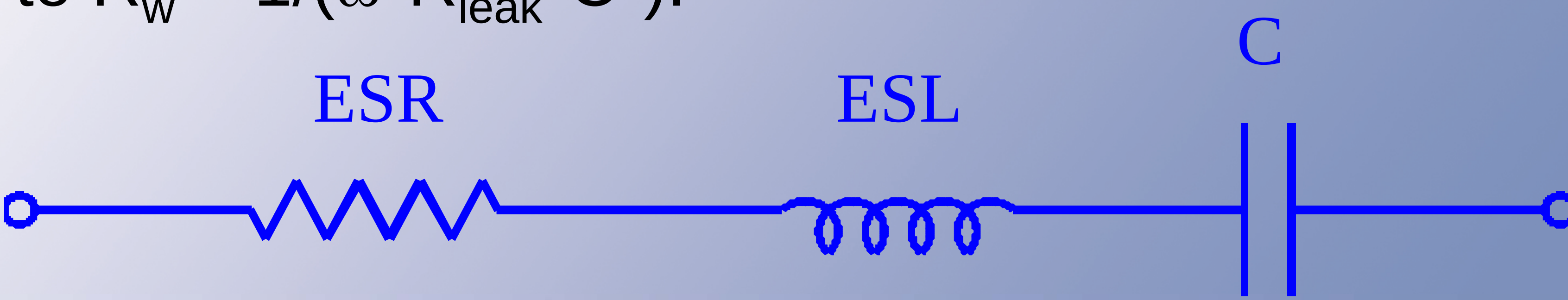
Circuit Model

- This looks complicated, but is easy to simplify because we expect a very long "leakage time constant" $R_{leak}C$.
- If $R_{leak}C$ is a long time, the ratio $R_{leak}C/T$ is large.
- In turn, the quantity $\omega^2 R_{leak}^2 C^2 \gg 1$.



Circuit Model

- Define equivalent series inductance, ESL, equal to L_w.
- Define *equivalent series resistance*, ESR, equal to $R_w + 1/(\omega^2 R_{leak}^2 C^2)$:



Implications

- This is a resonant circuit.
- Below resonance, the reactance is negative (we have C).
- Above resonance, the reactance is positive -- we have L!
- This is the *standard model of a capacitor*.

Some Concerns

- To get capacitive filtering, we need to operate below the *self-resonant frequency*, $f_r = 1/[2\pi\sqrt{(ESL)C}]$.
- This is nontrivial. For example, 20 nH and 500 uF yields 50 kHz as an upper limit.

Summary So Far

- ESL → related to the geometry (wires, layout, internal construction)
- ESR → wire effects plus transformed leakage resistance
- C → the internal charge storage $\varepsilon A/d$.

Behavior

- Consider |Z| and ∠Z.
- Well below self-resonance, the impedance falls with frequency, and the angle is -90°.
- Well above self-resonance, the impedance *rises* with frequency, and the angle is +90°.
- At self-resonance, Z = ESR, and the angle is zero.

Behavior

- The very best capacitors have low ESL and ESR values, and show a sharp self-resonance.
- Capacitors with higher ESR will show a shallower resonance effect, with a gradual change in the angle.

Some Concerns

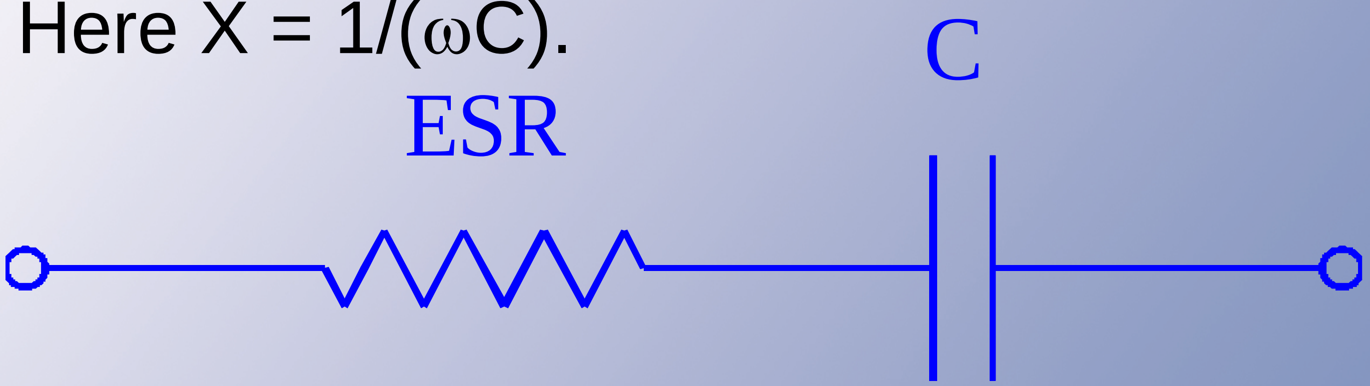
- From a source impedance perspective, we have |Z| that ultimately rises at high frequency.
- There is loss in the resistance.
- We want the resistive voltage drop to be negligible.

Finding ESR

- ESR comprises a leakage resistance effect plus series resistance of wires and materials.
- For simple dielectrics, we might estimate it with low wire resistance.
- Then ESR $\approx 1/(\omega^2 R_{leak} C^2)$.

Finding ESR

- We define the *dissipation factor*, *df*, as the ratio R/X for the series model (at low frequency). Here X = $1/(\omega C)$.



- This is df $=(ESR)\omega C \approx 1/(\omega R_{leak} C)$.
- The dissipation factor is also called the *loss tangent*, tan δ.

Finding ESR

- The loss tangent is a geometry-independent material property.
  - $C = \varepsilon A/d$
  - $R_{leak} = \rho d/A$
  - $R_{leak}C = \rho\varepsilon$ ← a material property
- For many good dielectric materials, the loss tangent is roughly constant with frequency.
- This allows us to say ESR $\approx (\tan \delta)/(\omega C)$.

Finding ESR

- More generally, ESR = $(\tan \delta)/(\omega C) + R_w$.
- For electrolytic capacitors, the connection resistance cannot be neglected, and ESR is more dominated by R_w.

Construction

- Most capacitors ultimately have two conducting surfaces and insulation in between.
- The *simple dielectric* construction is most direct, with clear plates and insulated spacers.

Simple Dielectric Materials

- Polymer films.
- Ceramics.
- Paper, mica, and other insulators.

- Ceramics for high ε. Others for low loss or low cost.

- Almost every planar insulation material has been tried (and sold).

Structure

- The planar structure might be flat (common with ceramics) or rolled (common with polymers or paper).
- Aluminum conductors are common.
- For polymer films the limits are on “thinness” of the films and conductors.
- *Multi-layer ceramic* capacitors place several layers in parallel.

Characteristics

- Simple dielectrics tend to follow the standard model very well.
- Voltage ratings are high.
- For polymers, ε is low (perhaps $2\varepsilon_0$ to $3\varepsilon_0$).
- For polymers, thin spacings but low capacitance per unit volume.

Characteristics

- Ceramic capacitors are built in simple dielectric form.
- For ceramics, ε up to $1000\varepsilon_0$.
- The spacing must be thicker, although voltage ratings are still limited (by the material strength).
- Often sensitive to moisture.
- Expensive in “large” values.

Characteristics

- Many polymers provide df < 0.01, or $\tan\delta < 1\%$, and sometimes below 0.1%.
- Ceramics tend to have $\tan\delta$ in the range of 1% to 5%.
- Define *quality factor* $Q = Z_c/ESR$, where Z_c is the characteristic impedance $Z_c = \sqrt{(ESL/C)}$.
- Most simple dielectrics have high Q.

Electrolytics

- What does it take to get high C values in small packages?
  - Thin spacings
  - Large areas
  - High ε values
- How to accomplish all of this?

Electrolytics

- Certain metals have interesting insulating oxides.
- Classic example: alumina. This is a very high-quality oxide with good electrical properties.
- It forms a uniform protective layer on the surface of pure aluminum.

Electrolytics

- Most other oxides might shrink (to form exposed cracks) or grow (to lose contact with the metal).
- Aluminum and tantalum have the best oxide properties from an electrical standpoint.

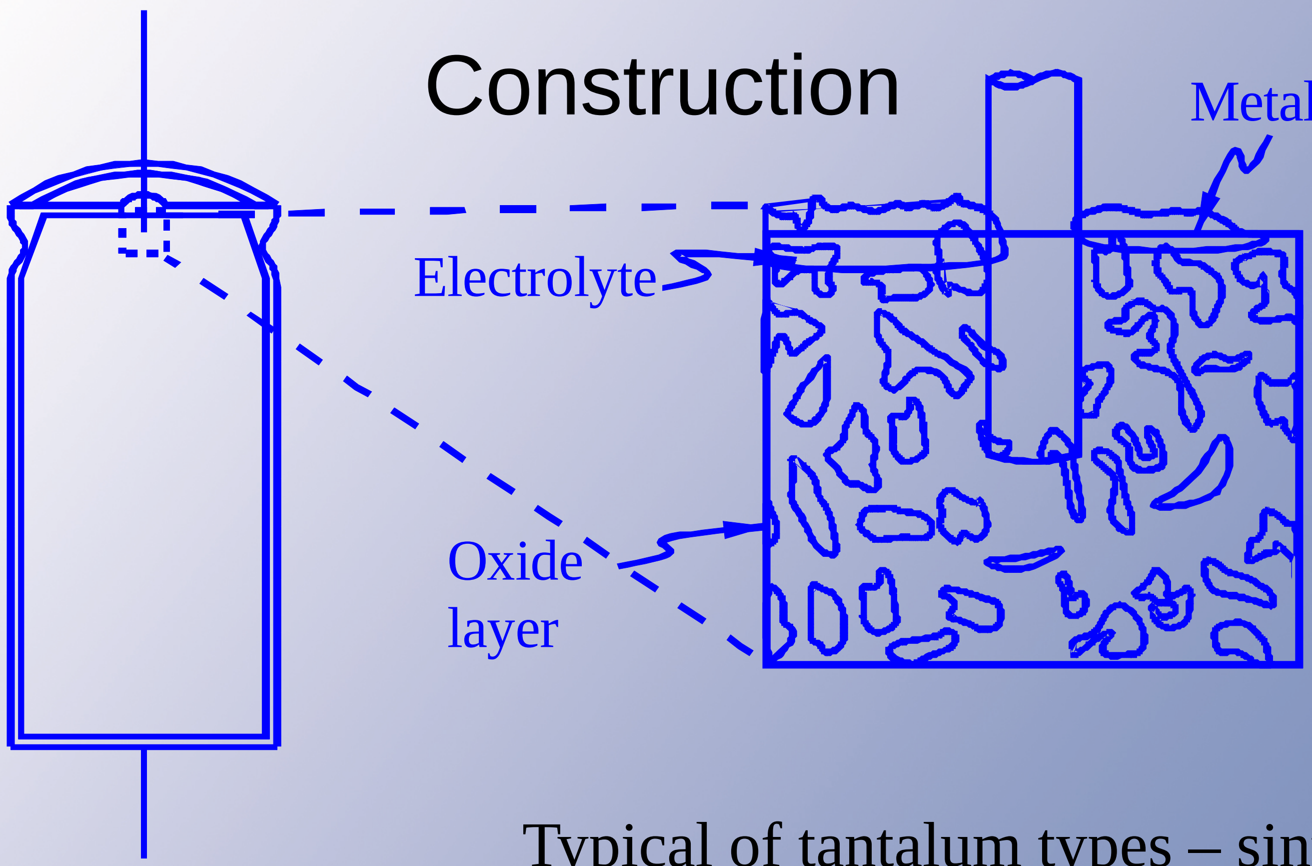
Electrolytics

- Concept:
  - Etch material, or start with a fine powder, to get high surface area.
  - Electroplate the material with its own oxide.
  - Create electrical connections to the material and to the outer side of its oxide.
- The connections are the hard part.

Electrolytics

- Problem: If we can plate the material, we can "unplate" it, too.
- Electrolytics have polarity.
- Reverse polarity will degrade the oxide layer and cause short circuit failure.

Construction



Typical of tantalum types – sintered.

Expanded view shows oxidized metal slug. Voids must contain electrolyte.

Electrolytics

- Connections:
  - The metal is connected directly to a wire lead.
  - A sheet is etched for surface area, or
  - The powder is sintered to form connections among the particles.
- The other side could be connected with a liquid or solid conductor: the electrolyte.

Electrolytics

- The electrolyte that makes the second contact can be wet (often water-based) or dry (such as the manganese dioxide material used in dry cell batteries).
- The ESR values are higher for a given C than simple dielectrics because of the higher effective “wire” resistance.

Electrolytics

- Since the electrolyte introduces series resistance, the ESR is nearly constant with frequency.
- Electrolytics tend to have “short” failure modes: polarity reversal or heating will be concentrated at the thinnest part of the oxide, and it will degrade and short circuit.

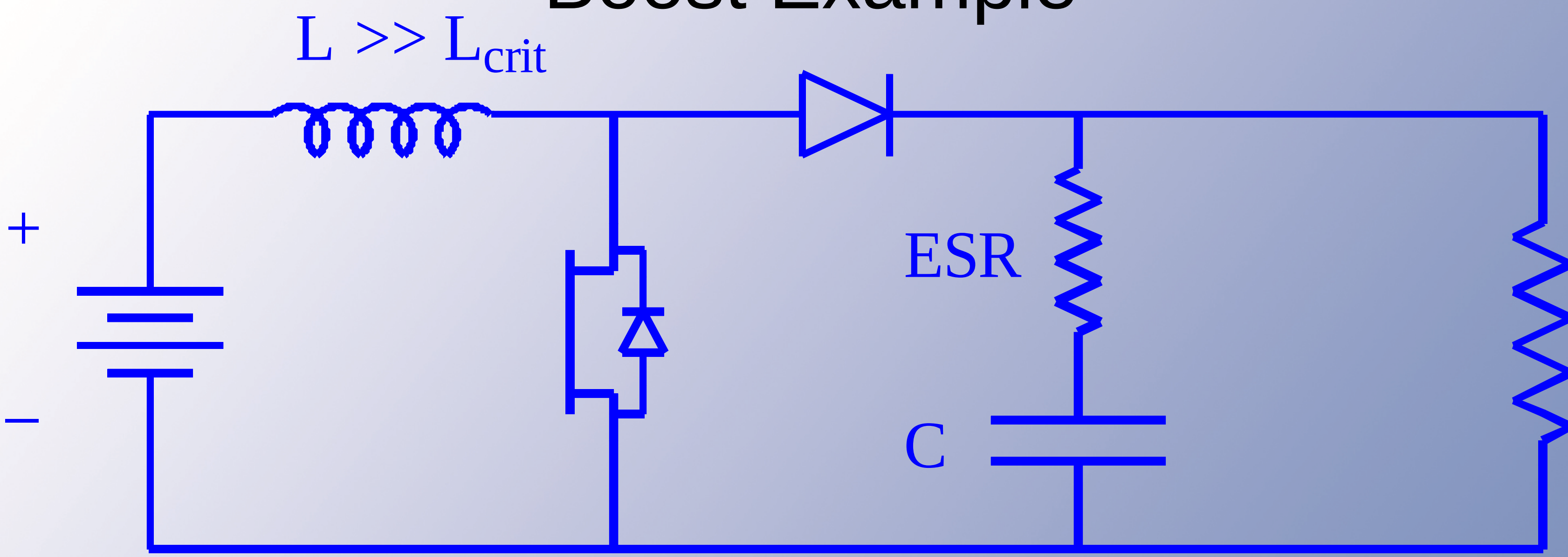
Converter Effects

- We need to choose a capacitor with self-resonance well above the strong unwanted frequencies to be filtered.
- Below self-resonance, the circuit model is the ESR in series with C.

Sample Case

- Consider a boost converter.
- The output voltage ought to be fixed.
- Notice that the series ESR does not alter a key fact: $\langle i_C \rangle = 0$.
- In reality, leakage does allow small current flow (mA).

Boost Example



- Relationships are the same, but output ripple is different.

Converter Relationships

- Transistor voltage: q_2V_{out}. The average is $D_2V_{out} = V_{in}$.
- Diode current: q_2I_L. The average is $D_2I_L = I_{out}$.
- The ESR does not change the general behavior.

Ripple Effects

- For ripple, however, the output is now $v_C + v_{ESR} = v_{out}$.
- We expect v_C to be triangular as before, but what about v_{ESR}?
- With the diode off, i_{out} flows out of the capacitor; v_C falls and v_{ESR} makes v_{out} lower.

Ripple Effects

- Diode off:

$$i_C = -I_{out}$$
$$v_C \rightarrow \text{falling}$$
$$v_{out} = v_C - I_{out}\,ESR$$

- Diode on:

$$i_C = I_L - I_{out}$$
$$v_C \rightarrow \text{rising}$$
$$v_{out} = v_C + (I_L - I_{out})\,ESR$$

Ripple Effects

- With the diode on, current I_L - i_{out} flows into the capacitor.
- The voltage v_C rises, and v_{out} is higher than v_C.
- What is the change?

$\Delta v_{out} = \Delta v_C + \Delta v_{ESR}$.

Ripple Effects

- Δv_C:
  - Diode off, $i_C = -i_{out} = C\, dv_C/dt$, $\Delta v_C = i_{out} D_1 T/C$.
- Δv_{ESR}:
  - $\Delta v_{ESR} = (I_L - i_{out})ESR - (-i_{out})ESR$
  - $\Delta v_{ESR} = I_L\, ESR$

Ideal Case

- If R_w is small, ESR = $(\tan \delta)/(\omega C)$.
- The total change, when the ESR value can be found from $\tan \delta$, is
$\Delta v_{out} = i_{out} D_1 T/C + I_L \tan \delta/(\omega C)$.
- This reduces to
$\Delta v_{out} = i_{out} D_1 T/C + (2\pi/D_2) i_{out} (\tan \delta) T/C$.
- The change is still proportional to i_{out} T/C, but is larger.

Nonideal case

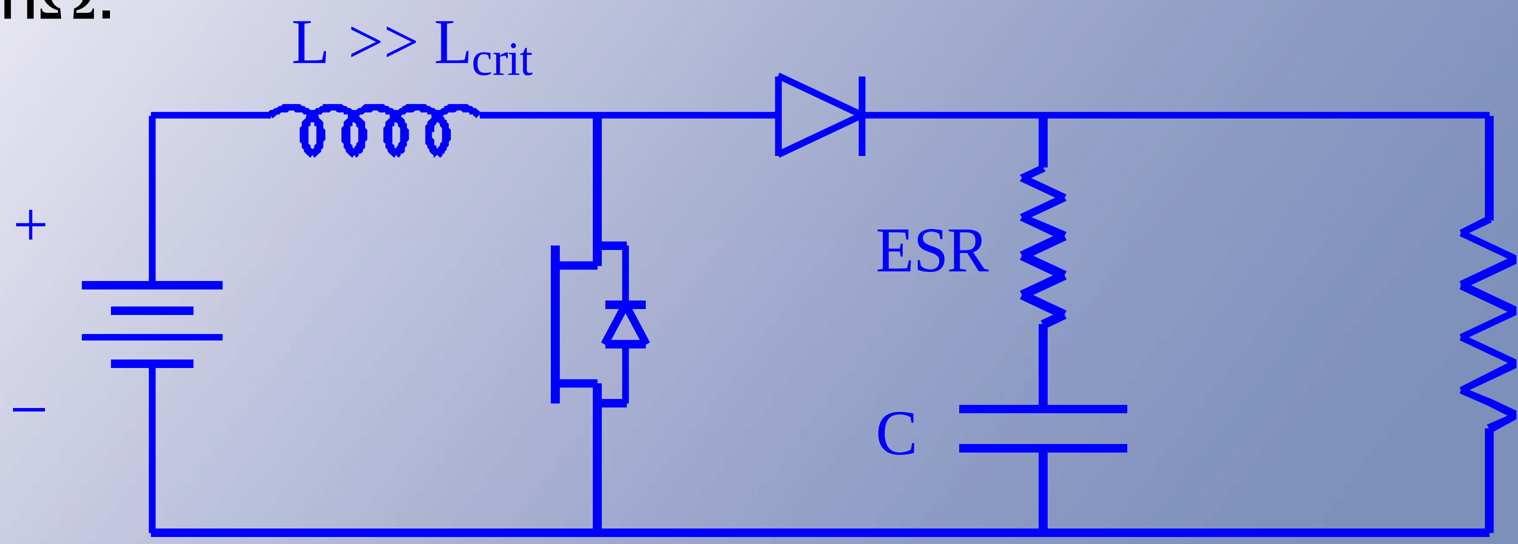
- If R_w is not small, the ESR jump also includes a term $R_w I_L$ that is independent of frequency.
- However, in electrolytics, the electrolyte resistance depends on area, so higher C generally gives lower R_w.
- In any case, ripple is proportional to 1/C.

Nonideal Case

- At the highest current levels (especially at low voltage), ESR jump dominates the ripple.
- The capacitor in a 5 V or lower converter is often selected based on ESR, not really on its value of C.

Numerical Example

- A 12 V to 48 V boost converter, 200 W, 50 kHz switching.
- Find C to provide ±0.5% ripple, given that tan δ = 0.20 and that R_w gives a minimum ESR of 10 mΩ.



Change

- The inductor current is 16 2/3 A. The output current is 4 1/6 A.
- D_1 is 0.75 and D_2 is 0.25.
- Ripple should not exceed 0.48 V.
- The change Δv_C is i_{out} D_1 T/C = 62.5×10^{-6}/C.
- With no ESR, C > 130 uF works.

With ESR

- The ESR value: 0.01+ tan $\delta/(\omega C)$.
- The change is 16 2/3 A times this, or 0.1667 V + 10.6 x 10^{-6}/C.
- The total required is now 233 uF, almost double.
- Notice that ripple below 0.35% cannot be achieved, because of R_w.

Summary So Far

- Real capacitors have a self-resonant frequency, and are useful below this frequency.
- In a power converter, the unwanted (ripple) frequencies determine this usefulness.
- We must include ESR to get accurate results.

Summary So Far

- ESR voltage drop adds a square wave ripple on top of the usual triangular ripple.
- This is called the ESR jump.
- At high currents and low voltages, ESR jump can dominate ripple.
- ESR is linked to both a loss tangent and series resistance effects.

Wire Resistance

- Wires have resistance, with $R = \rho l/A$ (ρ -- resistivity, l -- length).
- The power loss per unit volume of material is $i_{RMS}{}^2 R/(lA) = i_{RMS}{}^2 \rho/A^2$.
- Current per unit area is current density, J. The loss per unit volume is ρJ^2.

Wire Resistance

- We would expect some limit on loss per unit volume.
- Perhaps a block of metal can dissipate 1 W/cm^3 without problems.
- This implies a limit on current density.
- For copper, $\rho = 1.724 \times 10^{-8}\ \Omega$-m.
- At 1 W/cm^3, the limit is $7.62 \times 10^6\ A/m^2$.

Current Density Limits

- In power electronics practice, it is usual to limit current densities to the range of 10^6 to 10^7 A/m^2, or 100 to 1000 A/cm^2.
- The higher values apply when heat is less important.

Wire Size

- Consider a #18 AWG wire, which has a diameter very close to 1 mm.
- The cross section area is πr^2 = 0.78 mm^2.
- At 1000 A/cm^2, this implies a limit of 7.8 A.
- Sure enough, real products never push above 10 A in a #18 wire, and often closer to 5 A.

Wire Size

- In the U.S., we use "American Wire Gauge," which is a complicated logarithmic scale.
- There is an easy way to remember it, though:
  - #18 wire is very close to 1 mm diameter.
  - Every 3 steps in gauge yields a factor of 2 in area.
  - #15 is twice as big as #18, #12 is 4x, etc.

Current Rating

- If #18 can carry 5 A easily, we expect #12 to carry 20 A.
- Sure enough, #12 AWG is used for 20 A house circuits.
- #24 wire can carry about 1.2 A without trouble.
- A table in the book gives sizes and current capacity examples.

Current Density Limits

- Since we seek efficiency, lower current densities are good.
- Example: #22 AWG wire carrying 3.26 A (1000 A/cm^2) has loss (per meter of length) of 0.57 W/m.
- This might seem low, but #18 loses only 0.23 W/m for this length.

Current Density Limits

- #22 with 10 A loses 5.4 W/m -- and gets hot.
- Voltage drop can also be an issue. Consider a 5 V, 200 W supply -- 5 V and 40 A.
- Even at 500 A/cm^2 (#8 wire), the drop is 84 mV/m.

Drop Issues

- Example: Use #8 wire to connect a 5 V supply to a 200 W load, 25 cm away.
- Total wire length: 0.5 m.
- Drop: 42 mV (0.84%).
- Loss: 1.7 W -- almost 1% of output.

Drop Issues

- Think about 2 V at 20 A (not an uncommon microcomputer supply).
- How big a wire, how much drop?

Drop Issues

- Perhaps we can use #12 AWG?
- The resistance is 5.3 mΩ/m.
- At 20 A, the voltage drop is 0.11 V/m.
- 10 cm of wire yields 0.011 V drop, which is more than 0.5% of the intended 2 V.

Drop Issues

- It is also worth considering the inductance effect.
- A 10 cm wire has about 50 nH of inductance.
- A current change of 0 A to 20 A in 10ns will yield 100 V induced along the wire!
- Even 1 nH would yield 2 V drop.

Drop Issues

- Fast transient loads with low voltage supply levels cannot really be supplied through conventional wires.
- Small capacitors must be present right at the load.
- Even then, stray inductance is a problem.

Thermal Issues

- Loss leads to heat generation.
- Nearly all metals have a resistivity that rises with temperature. This is especially important for resistor design.

Temperature Coefficient

- Example case: Copper.
- The resistivity increases by +0.39% for each 1° C (1 K) increase.
- This seems small, but consider that a 20 K rise gives a 7.8% increase. Not good for resistors if precise values are desired.

Application Example

- An interesting effect occurs in heaters or lamps.
- For example, if we want an oven heating element at 350°C, made from copper, the "hot" resistance is 2.29 times that at 20°C.
- Sizing is a challenge.

Application Example

- Now, set it up for 4000 W at 230V_{RMS}. This requires 13.2 Ω at the high temperature.
- The current is 17.4 A.
- Copper at the low temperature would have resistance of only 5.78 Ω, and I = 39.8 A. *Inrush*!

Frequency Issue

- Internal self-inductance forces the current toward the surface of a conductor as frequency increases.
- The "skin depth" is $d = \sqrt{[2\rho/(\omega\mu)]}$, or $(0.166)\sqrt{[1/\omega]}$ (in meters), for copper.

Frequency Issue

- At 50 kHz, this is 0.3 mm -- enough to matter for wire bigger than about #22.
- The net effect is an increase of resistance with $\omega^{-1/2}$.
- Litz wire and thin bus bar can avoid this. (Why?)

Resistance

- To avoid large changes, resistors should be made of materials with low thermal coefficients of resistivity.
- Classic example: nichrome (80% nickel, 20% chromium) with 0.01% change per degree.

Resistance

- Nichrome is very widely used in heating applications.
- For our oven, the change is only 3.3% over the full range, and the inrush problem is avoided.

Resistors -- Points

- We want to make resistors from thermally constant materials.
- Resistors (especially those wound with wire) have inductance and capacitive effects.
- For “film” resistors, the frequency effects are small.

Resistors -- Points

- For wirewound resistors, inductive effects can be very large -- perhaps 10 nH for each cm of total wire length.
- We could wind them with dual opposite wires. This cancels much of the inductance.